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The Influence of Test Colors on the Color Rendering Index Maximization in a Three Component Spectral Power Distribution

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THE INFLUENCE OF TEST COLORS ON THE COLOR RENDERING INDEX MAXIMIZATION IN A THREE COMPONENT SPECTRAL POWER DISTRIBUTION

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ABSTRACT

This thesis describes the technique of principle component analysis as applied to determining if a combination of three characteristic vectors would account for an underlying structure in the eight color test objects specified by the CIE for calculation of the color rendering index (CRI). Anticipated was that three vectors would account for 100 % of the variance in the eight test objects; however it was determined that three vectors account for only 96 % of the variance.

Since the analysis of the reflectance functions showed no dependence within the functions a 3^3 factorial experiment was conducted. Its purpose was to determine if the point of maximization of CRI in a three component spectral power distribution would be at 610, 540, and 450 nm when using metameric gray objects. This was postulated by Mr. Thornton in his paper "Luminosity and the Color Rendering Capability of White Light". The analysis of the resulting response surface indicated that the maximization point was not the same but if a valid point exists it lies outside the experimental boundaries of the experiment.

INTRODUCTION

Color in a perceived sense is determined by four factors, the spectral power distribution of the illuminating source, the spectral reflectance function of the object, the surround, and the human visual response. A general expression for the effect of the illuminant on a set of perceived object colors in comparison with the perceived color under a reference illuminant is provided by the color rendering index (CRI). Adopted in 1964 by the International Commission on Illumination (CIE) was the CIE Method of Measuring and Specifying Color Rendering Properties of Light Sources.[1] Use of this method produces a color rendering index, R, which is based on the amount of colorimetric shift of a set of test object colors under a reference illuminant in comparison to their color under a test illuminant of identical or nearly identical chromaticity. Agreed upon in this standard is that the CIE method be considered fundamental for appraising the color rendering properties of light sources and be recommended for design and testing of lamps.

In the paper "Luminosity and the Color Rendering Capability of White Light" it was shown that when luminosity and CRI were used as criteria, some optimized white light spectral power distribution (SPD) would significantly improve the light.[2] The study concluded that an additive combination of three spectral lines near 450,540, and 610 nm approximated the white light SPD necessary to maximize luminous efficiency and CRI. The maximum CRI of a three component lamp with a correlated color temperature of 6800 K and the CIE illuminant C as a reference was 80. Subsequent papers have shown an improvement of this figure to 94 when the three component distribution was combined with a continuous spectrum.[3]

Many questions have arisen concerning the adequateness of the original CIE set of eight color test samples for measurement of the general CRI. Nayatani et. al. showed that because of deficiencies in the number and choice of the test samples involved it is possible to derrive, for a given reference illuminant, an infinite number of test illuminants with a different SPD that all have perfect color rendering indices.[4] In agreeing with this view Dr. Wyszecki, of the National Research Council of Canada, has suggested the use of metameric gray object colors generated statistically so as to be non-linearly related in assessing the color matching properties of lamps.[5,6] Nayatani and Takahama have confirmed the effectiveness of using the twelve metameric grays derrived by Dr. Wyszecki in making an assessment of this type.[7]

Although the use of properly chosen line spectra can provide a wide gamut of colors these same spectra could be disastrous in reproducing certain colors. Since the CRI was shown to increase even more when combined SPDs were used it would be beneficial to know if the maximum of the three line spectra could be accounted for by an interrelationship in the test samples. If the samples are interrelated and can be represented by three functions it is highly probable that these three functions would be the cause of maximization of CRI at wavelengths of 450, 540, and 610 nm. Determination of the existence of any mathematical relationship among the eight CIE test objects is the intent of Phase I of this research.

In Phase II the research has two possible paths. Mr. Thornton postulated that regardless of the type of test objects used the CRI would maximize at the wavelengths he had determined. Should the test objects prove to be non interrelated the metameric grays would be used in a factorial experimental design to determine if the CRI maximization occurrs at the same point. The second path would be if the test objects prove to be related, the factorial experiment would then be aimed at determining a point of maximization.

COLORIMETRIC MEASUREMENT

Prior to any discussion of color rendering index one must be familiar with the procedures involved in the measurement and specification of color. Current specification in the CIE colorimetric system involves calculation of the X,Y,Z tristimulus values. To account for the human visual response the color matching functions for the CIE 1931 standard observer are used. When the standard observer views an illumination source the tristimulus values are given by:

$$X = k \sum P_\lambda \bar{x}_\lambda$$

$$Y = k \sum P_\lambda \bar{y}_\lambda$$

$$Z = k \sum P_\lambda \bar{z}_\lambda$$

where P_λ is the spectral distribution of the source and $\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$ are the color matching functions for the standard observer. The normalization constant k is usually chosen so that the resultant value of Y is 100.00.

An object color is similarly specified by:

$$X = k \sum p_\lambda H_\lambda \bar{x}_\lambda$$

$$Y = k \sum p_\lambda H_\lambda \bar{y}_\lambda$$

$$Z = k \sum p_\lambda H_\lambda \bar{z}_\lambda$$

where p_λ is the spectral reflectance of the object, H_λ is the spectral distribution of the illumination source. When using the 1931 CIE standard observer the normalization constant for object colors is defined by:

$$k = \frac{100}{\sum H_\lambda \bar{y}_\lambda}$$

These tristimulus values lead directly to the x,y,z chromaticity coordinates which are related through the following equations:

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

In order to provide a representation of the color diagram which is more perceptually uniform than the x,y diagram a transformation is made to the CIE 1960 UCS diagram by the following equations:

$$u = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v = \frac{6Y}{X + 15Y + 3Z} = \frac{6y}{-2x + 12y + 3}$$

$$W = 25Y^{1/3} - 17$$

COLOR RENDERING INDEX

In accordance with the CIE specification for the measurement of CRI the calculations were performed in this research using the following steps:

1) CALCULATION OF CIE 1931 TRISTIMULUS VALUES

Using spectroradiometric measurements of the test and reference illuminants with the spectral reflectances of the test objects, the CIE 1931 tristimulus values X,Y,Z and the chromaticity coordinates x,y of the illuminants and the test objects under each illuminant are calculated.

2) CIE 1960 UCS TRANSFORMATION

The CIE 1931 tristimulus values are transformed to the CIE 1960 UCS coordinates. If the test and reference illuminants are not of identical chromaticities, as is the case in this research, a transformation would be required to account for the adaptive color shift under the two illuminants.

3) CIE 1964 UNIFORM COLOR SPACE TRANSFORMATION

The u,v coordinates are transformed to the UCS system by the following formulas:

$$W^*_{r,i} = 25(Y_{r,i})^{1/3} - 17 \qquad W^*_{k,i} = 25(Y_{k,i})^{1/3} - 17$$

$$U^*_{r,i} = 13W^*_{r,i}(u_{r,i} - u_r) \qquad U^*_{k,i} = 13W^*_{k,i}(u_{k,i} - u_k)$$

$$V^*_{r,i} = 13W^*_{r,i}(v_{r,i} - v_r) \qquad V^*_{k,i} = 13W^*_{k,i}(u_{k,i} - u_k)$$

where the subscripts r and k represent the reference and test illuminants respectively and the subscript i represents a test object color.

4) COLOR SHIFT

The color shift due to the two illuminants is given by:

$$\Delta E_i = (U^*_{r,i} - U^*_{k,i})^2 + (V^*_{r,i} - V^*_{k,i})^2 + (W^*_{r,i} - W^*_{k,i})^2$$

5) COLOR RENDERING INDEX

The color rendering index is calculated from the ΔE_i as follows:

$$R_i = 100 - 4.6\Delta E_i$$

This results in a scaling such that 100 represents a reproduction that is equal to the reference and a value of 50 represents the CRI of a standard warm white flourescent tube. The actual value of CRI is calculated as the arithmetic average of the R_is of the test colors used.

EVALUATION PROGRAM

Vector Analysis

The method of characteristic vector analysis is ideally suited to the determination of a mathematical relationship among the eight color test objects specified by the CIE. In this case each function is treated as a vector with 95 variables being over the wavelength range of 360 - 830 nm. This method will determine if a linear combination of three characteristic vectors will account for all the variance among the eight test object colors. The hypothesis is that three vectors will account for all variance and therefore would have some relationship to the point of maximization of CRI at 450, 540, and 610 nm. The mathematical detail and an example are given in Appendix A along with the computer program documentation.

FACTORIAL EXPERIMENT

In order to answer the question of where the CRI maximizes with a set of metameric gray object colors it is necessary to generate a response surface or equation for the dependent variable, CRI, as a function of the independent variables, in this case wavelength bands. Since the CRI reportedly maximizes at 450, 540, and 610 nm these three wavelengths were chosen as the central treatment combination of a 3 factorial experimental design. That is to say three wavelengths in each of three regions. A wavelength range of ± 10 nm from each center point was assumed which resulted in 27 unique combinations of the three factors. Using CIE illuminant C as the reference the power levels at each combination must be calculated so as to match the tristimulus values of Illuminant C. This can be accomploshed using a set of linear equations.

$$P_{\lambda 1}\bar{x}_{\lambda 1} + P_{\lambda 2}\bar{x}_{\lambda 2} + P_{\lambda 3}\bar{x}_{\lambda 3} = 98.0716$$

$$P_{\lambda 1}\bar{y}_{\lambda 1} + P_{\lambda 2}\bar{y}_{\lambda 2} + P_{\lambda 3}\bar{y}_{\lambda 3} = 100.0000$$

$$P_{\lambda 1}\bar{z}_{\lambda 1} + P_{\lambda 2}\bar{z}_{\lambda 2} + P_{\lambda 3}\bar{z}_{\lambda 3} = 118.2253$$

where $\lambda 1$, $\lambda 2$, and $\lambda 3$ are the three wavelengths in the treatment combination and $\bar{x},\bar{y},\bar{z}$ are the spectral tristimulus values of the CIE 1931 standard observer. The constants are the tristimulus values of CIE Illuminant C with the CIE 1931 standard observer. The calculated power levels represent the power required at each wavelength in order to match the tristimulus values and chromaticity coordinates of CIE Illuminant C.

A regression analysis on the factorial experimental data will provide the desired equation of response, CRI as a function of the input wavelengths over the region bounded by the experiment. Taking the partial derivatives with respect to the CRI for each variable and setting them equal to zero will result in the computation of a point on the response surface that is either a maximum or minimum. The second partial will indicate whether we have a maximum or minimum.

EXPERIMENTAL RESULTS

REFLECTANCE FUNCTION INTERDEPENDENCE

The CIE reflectance functions specified for use in the CRI calculation are given in Appendix B, Table 1. These reflectances were analysed using the characteristic vector analysis program in Appendix A. If the hypothesis that some combination of three unknown functions would account for the variance in the eight object colors then 100 % of the variance or trace of the variance-covariance matrix would have to be accounted for by three characteristic vectors. The results of these analyses given in Table 1 clearly indicate that three vectors account for only 96 % of the variance regardless of the base of the input data. That is either density or reflectance. These three vectors are illustrated in Figure 1. Figures 2 to 9 illustrate the real reflectance functions and the reconstructed reflectance functions based on the three characteristic vectors. Clearly with only 96 % of the variance accounted for the hypothesis must be rejected. Tables of the characteristic vectors, scalar multipliers, actual reflectance functions, the 3 and 7 component reconstructed functions and their associated errors are in Appendix B.

FACTORIAL EXPERIMENT

3-by-[illegible] MF

The results of the vector analysis indicated that the eight CIE/ CRI test object colors were not interrelated. Therefore the three factorial experiment was aimed at determining if the CRI, as determined by the twelve metameric gray objects, maximized

	REFLECTANCE			DENSITY		
COMPONENT	EIGENVALUE	PCT. TRACE	CUM. PCT.	EIGENVALUE	PCT. TRACE	CUM. PCT.
1	10.1515	66.9711	66.9711	95.7453	57.8507	57.8507
2	3.6879	24.3297	91.3009	54.0957	32.6854	90.5361
3	.8392	5.5362	96.8370	9.2875	5.6116	96.1478
4	.3641	2.4022	99.2392	5.6802	3.4321	99.5798
5	.0748	.4937	99.7328	.4480	.2707	99.8505
6	.0276	.1819	99.9147	.1704	.1030	99.9534
7	.0129	.0849	99.9996	.0761	.0460	99.9994

COMPONENTS FROM THE CHARACTERISTIC VECTOR ANALYSIS OF THE STANDARD CIE/CRI REFLECTANCE FUNCTIONS

TABLE 1

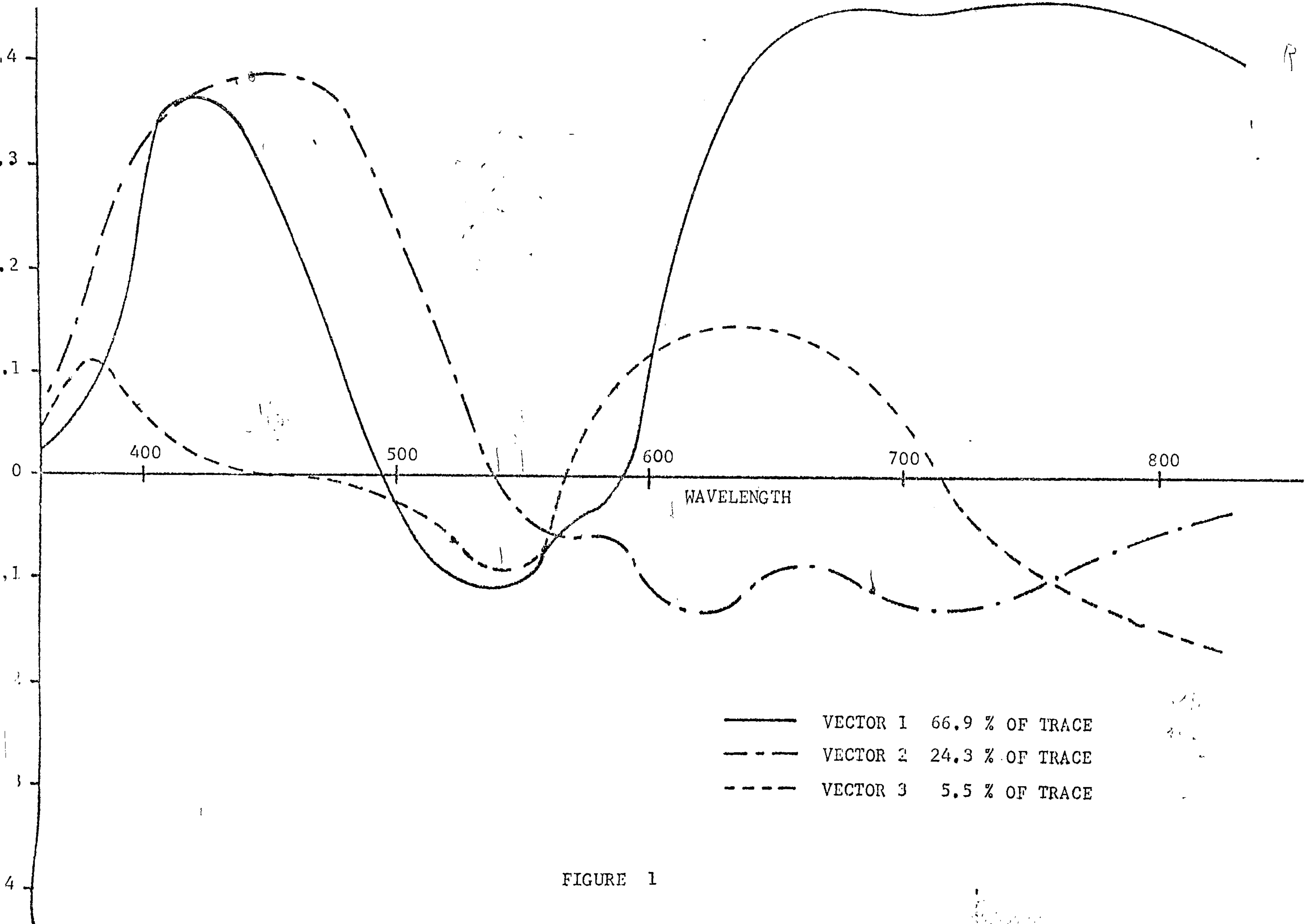

.4
.3
.2
.1
0
.1
4
400
500
600
700
800
WAVELENGTH
VECTOR 1 66.9 % OF TRACE
VECTOR 2 24.3 % OF TRACE
VECTOR 3 5.5 % OF TRACE

FIGURE 1

COLOR RENDERING INDEX TEST CHIP 1

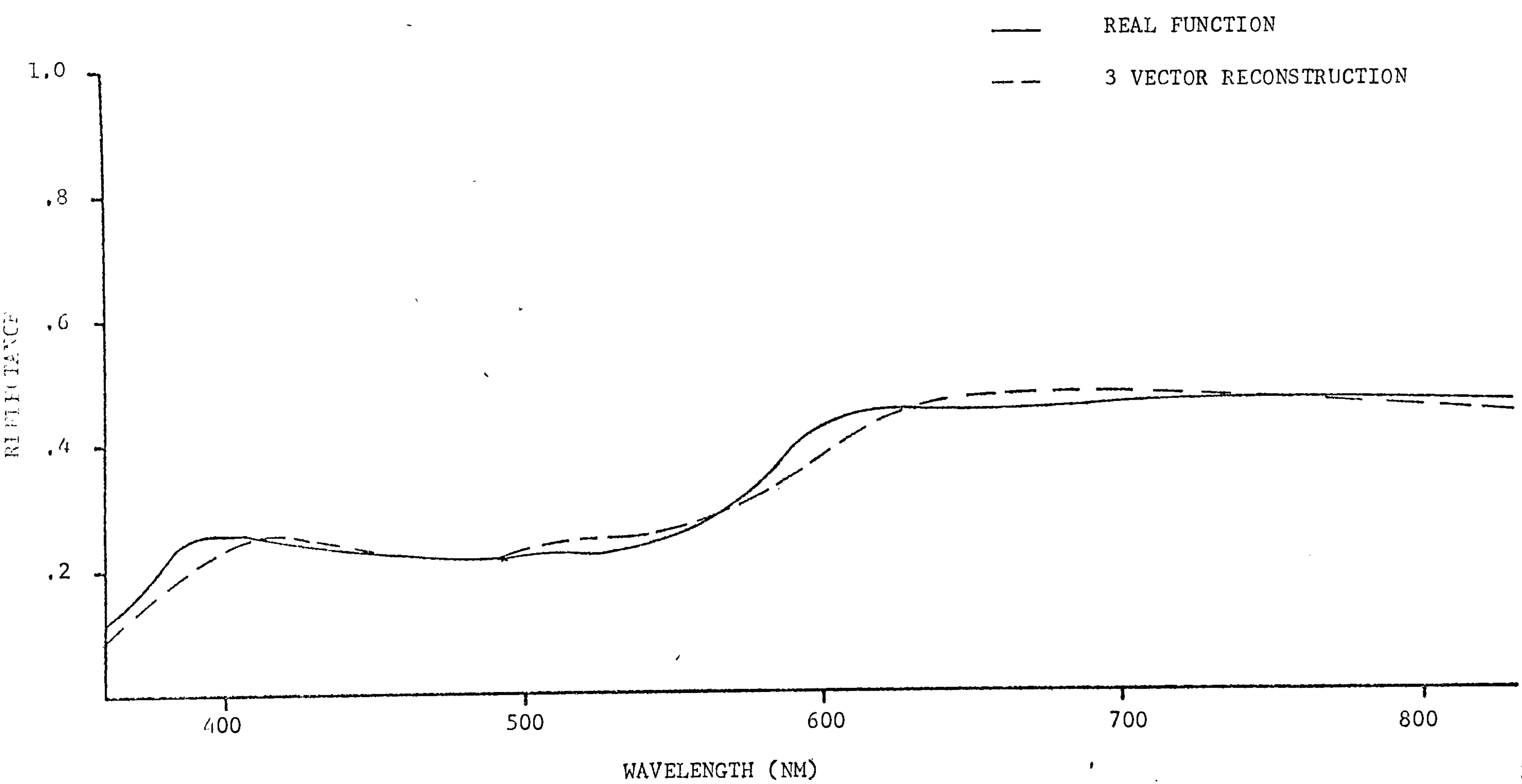
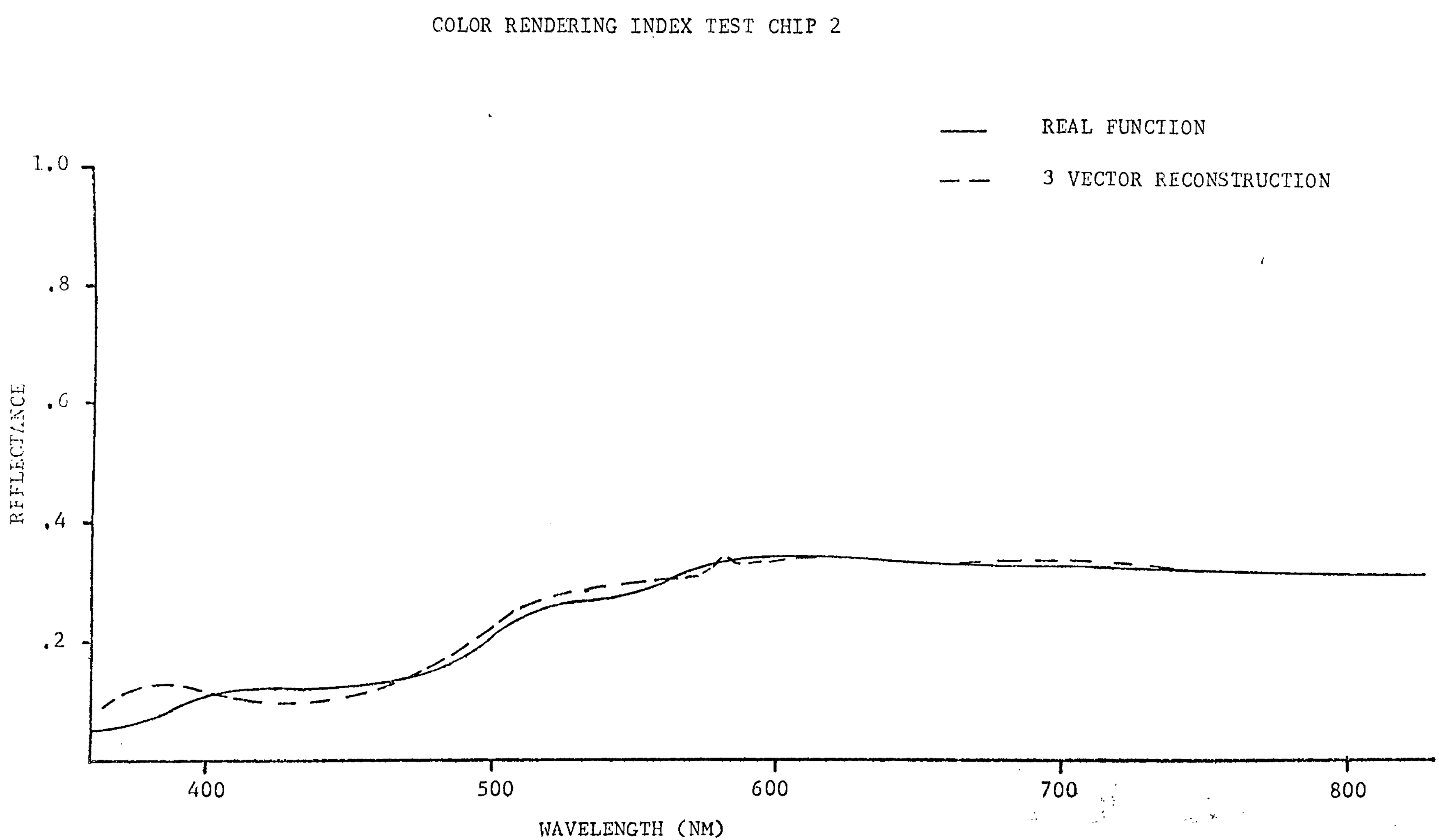


FIGURE 2



COLOR RENDERING INDEX TEST CHIP 2
REAL FUNCTION
3 VECTOR RECONSTRUCTION
REFLECTANCE
1.0
.8
.6
.4
.2
400
500
600
700
800
WAVELENGTH (NM)

FIGURE 3

COLOR RENDERING INDEX TEST CHIP 3

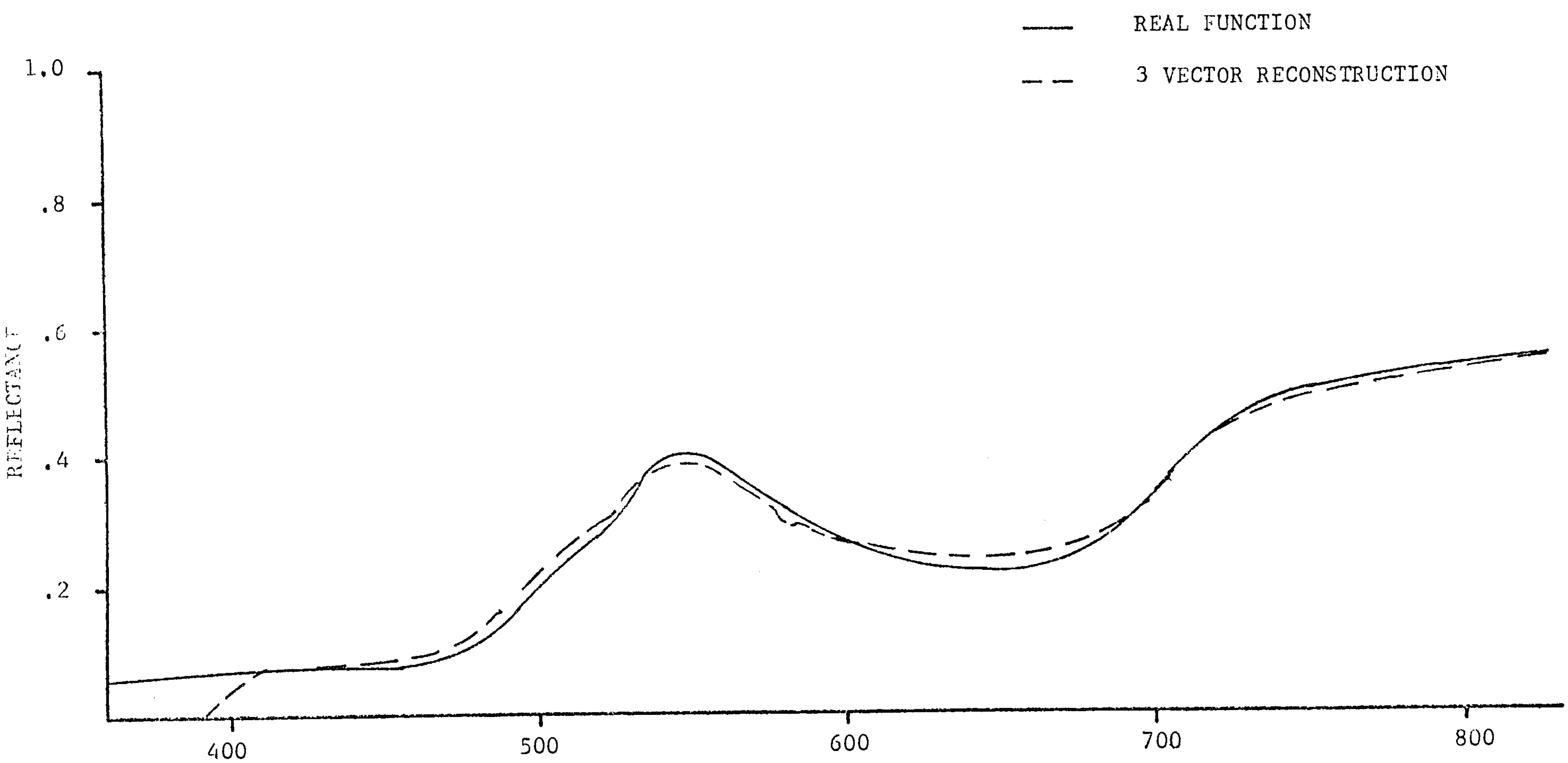


FIGURE 4

COLOR RENDERING INDEX TEST CHIP 4

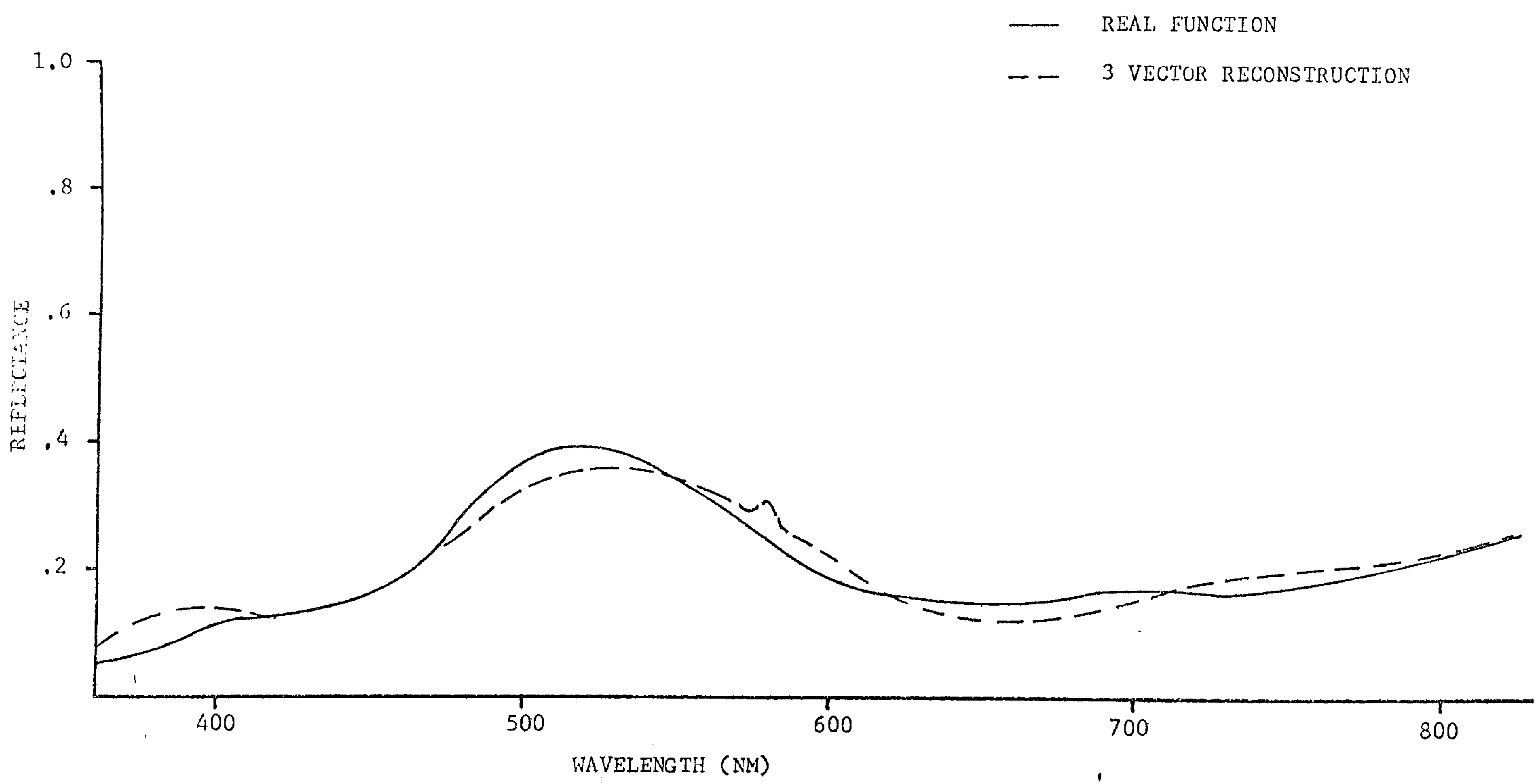


FIGURE 5

COLOR RENDERING INDEX TEST CHIP 5

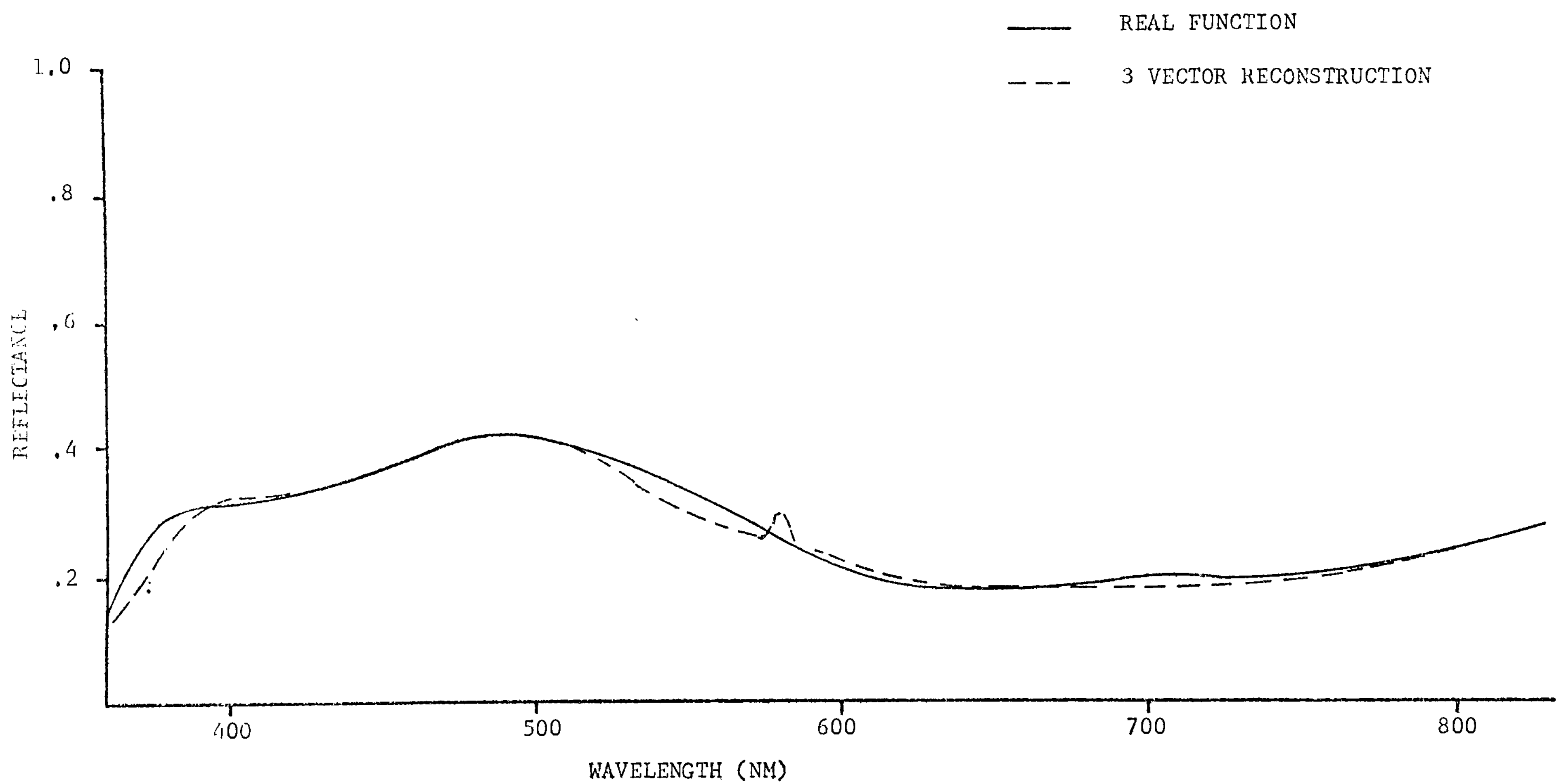
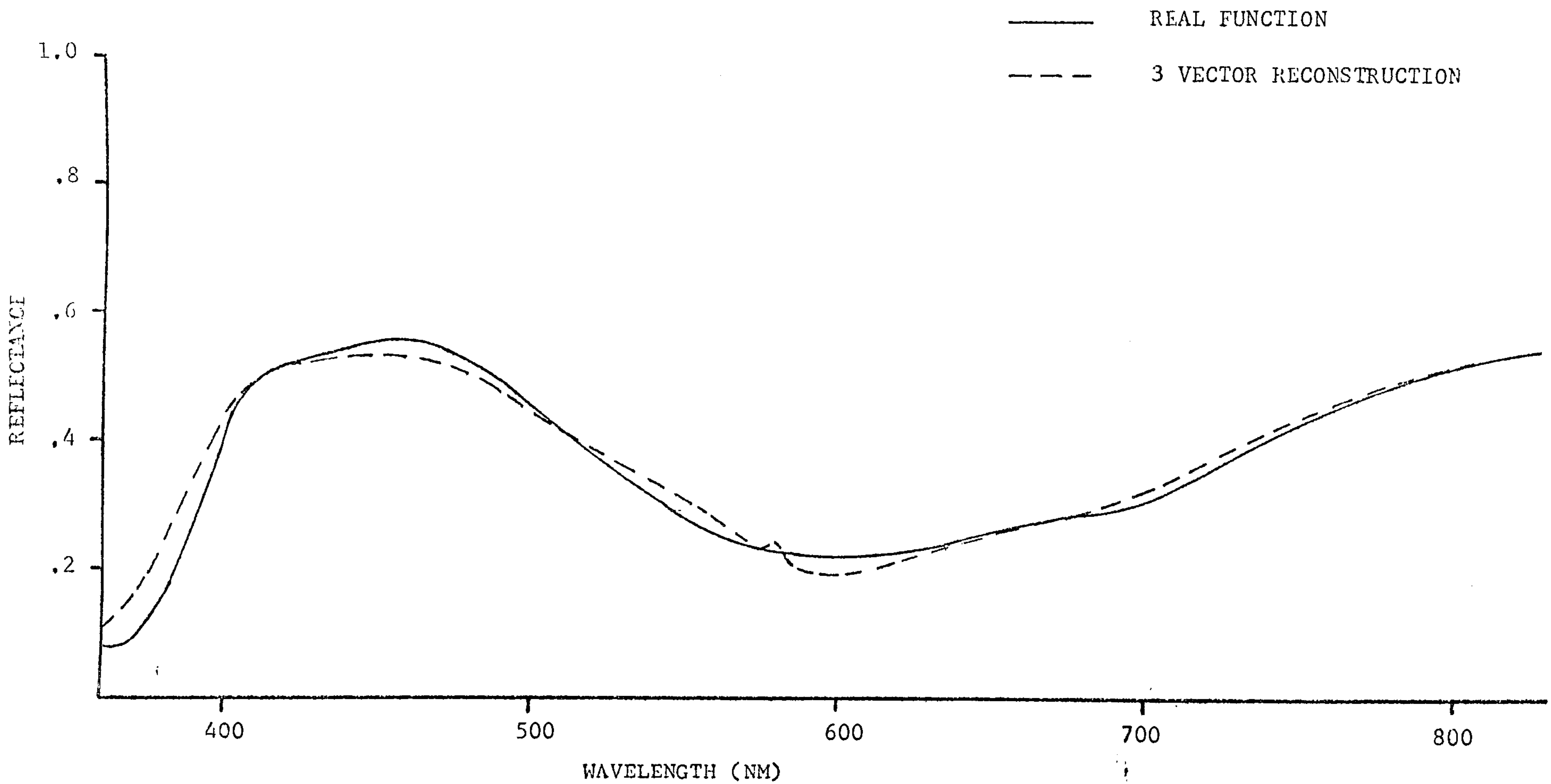


FIGURE 6

COLOR RENDERING INDEX TEST CHIP 6



COLOR RENDERING INDEX TEST CHIP 7

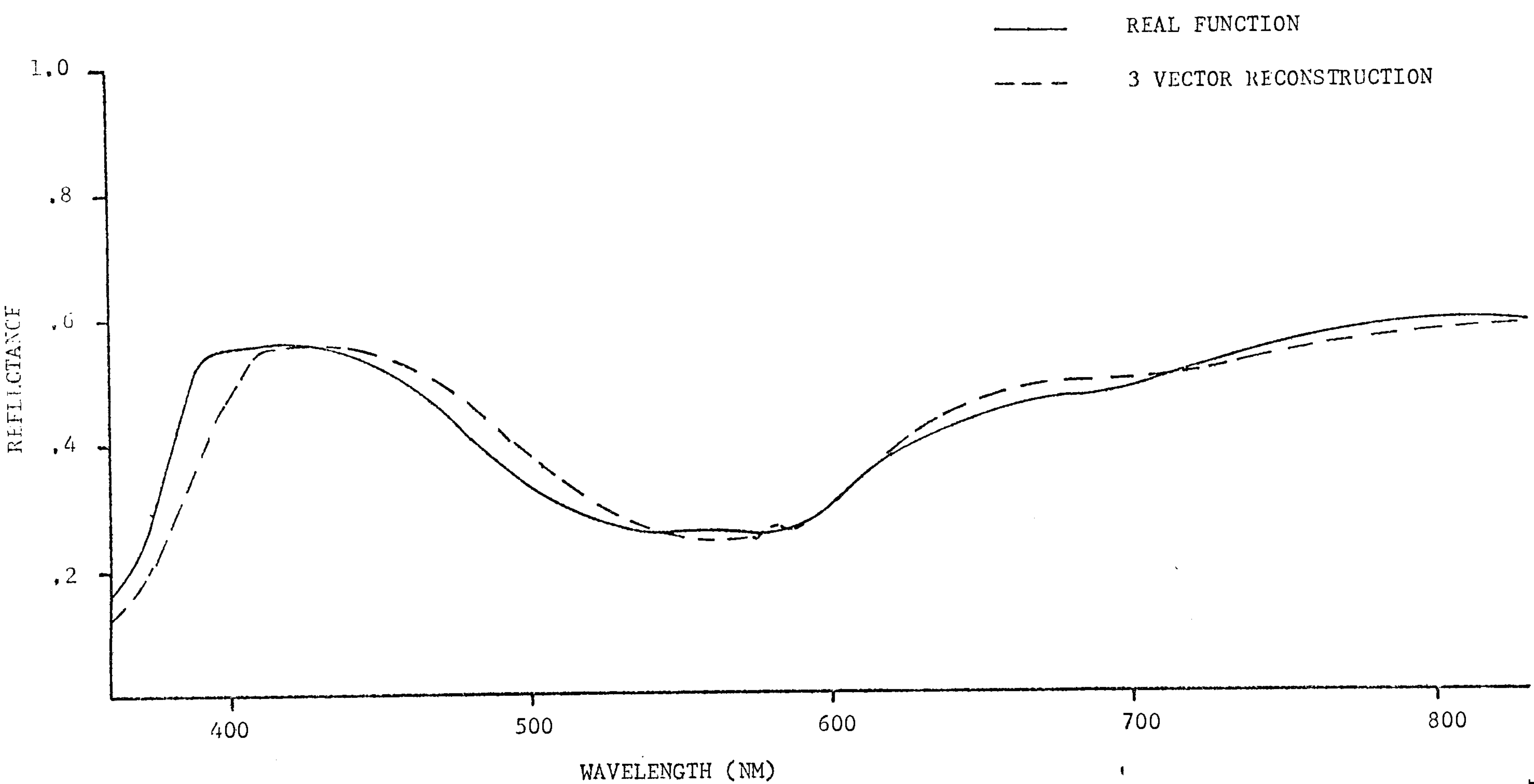
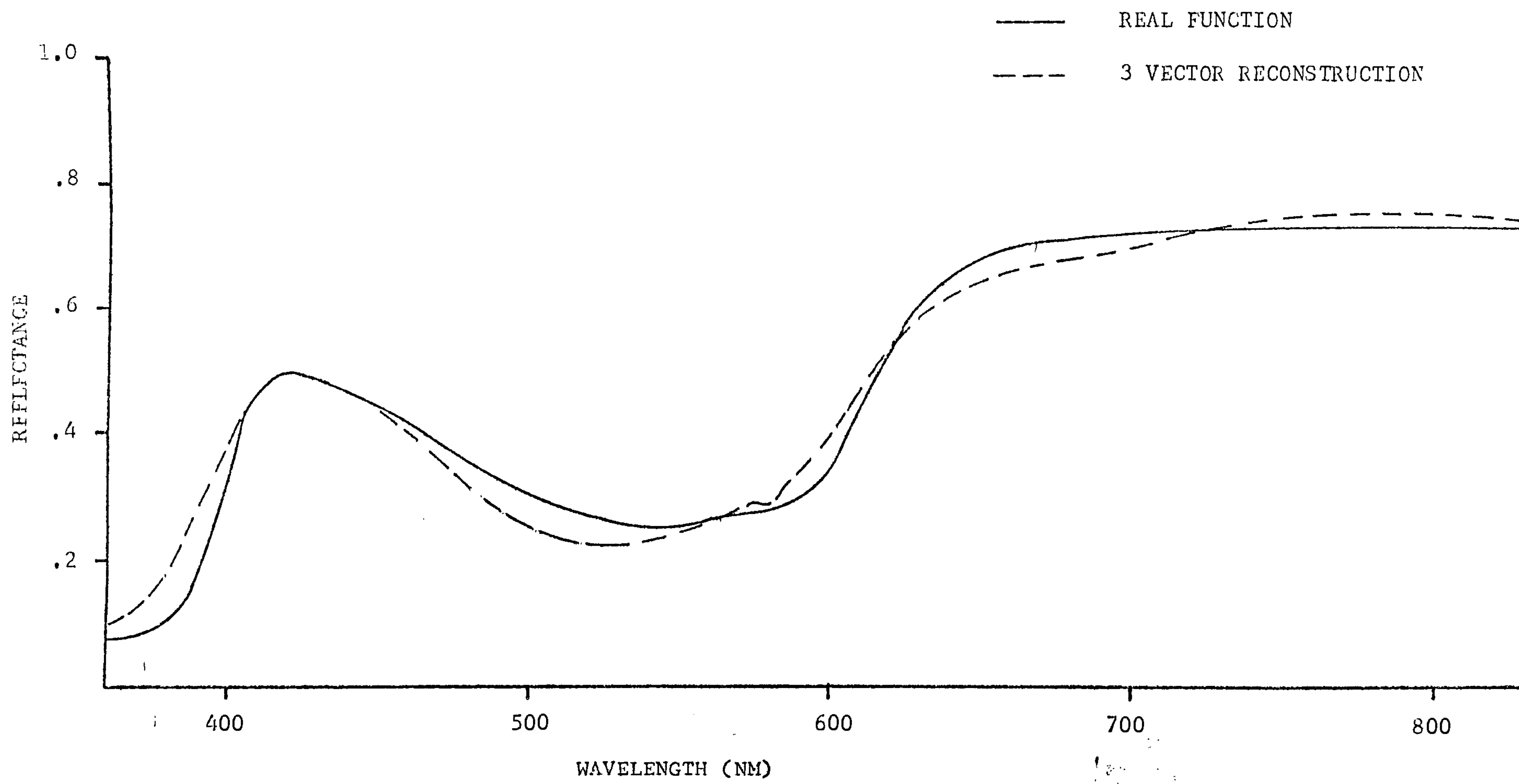


FIGURE 8

COLOR RENDERING INDEX TEST CHIP 8



AVERAGE OF COLOR RENDERING INDEX TEST CHIPS 1 - 8

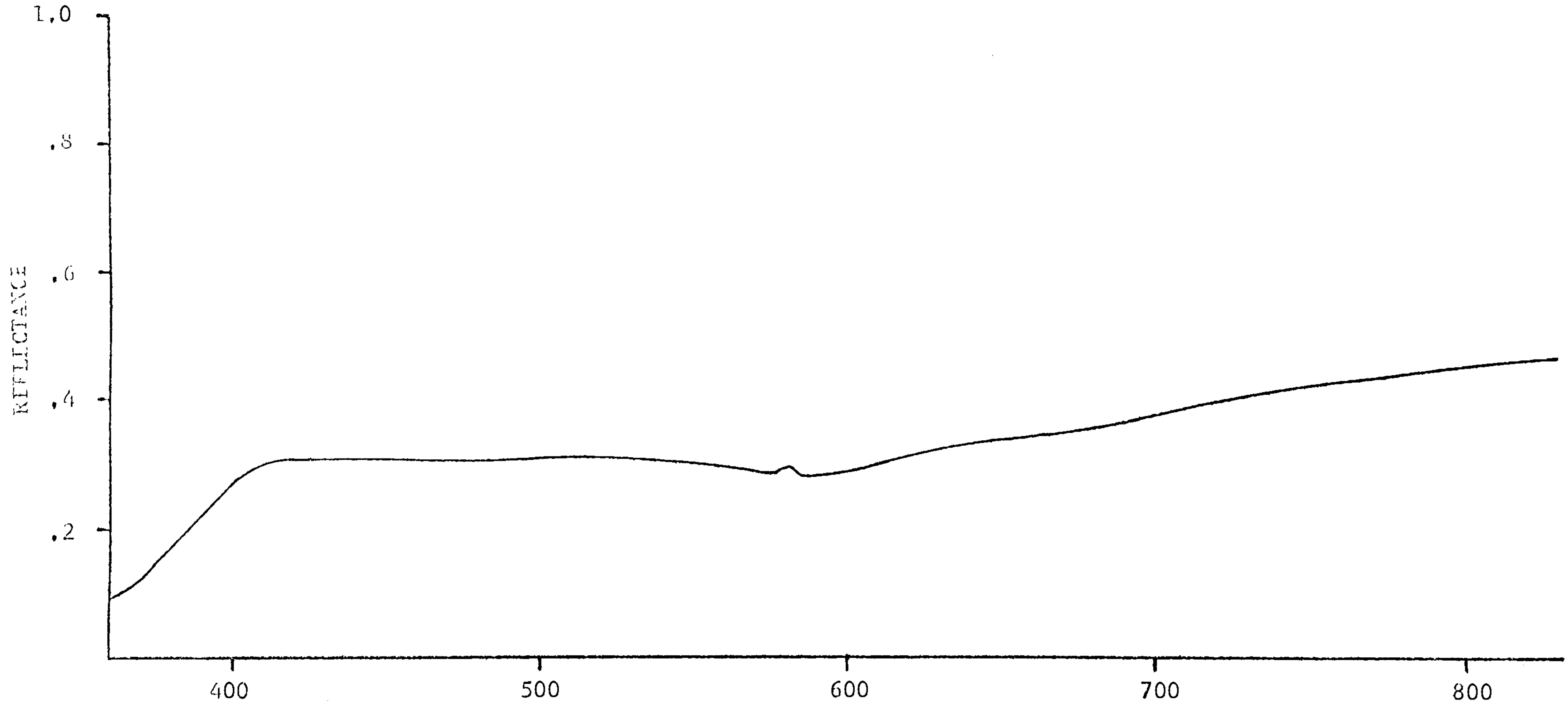


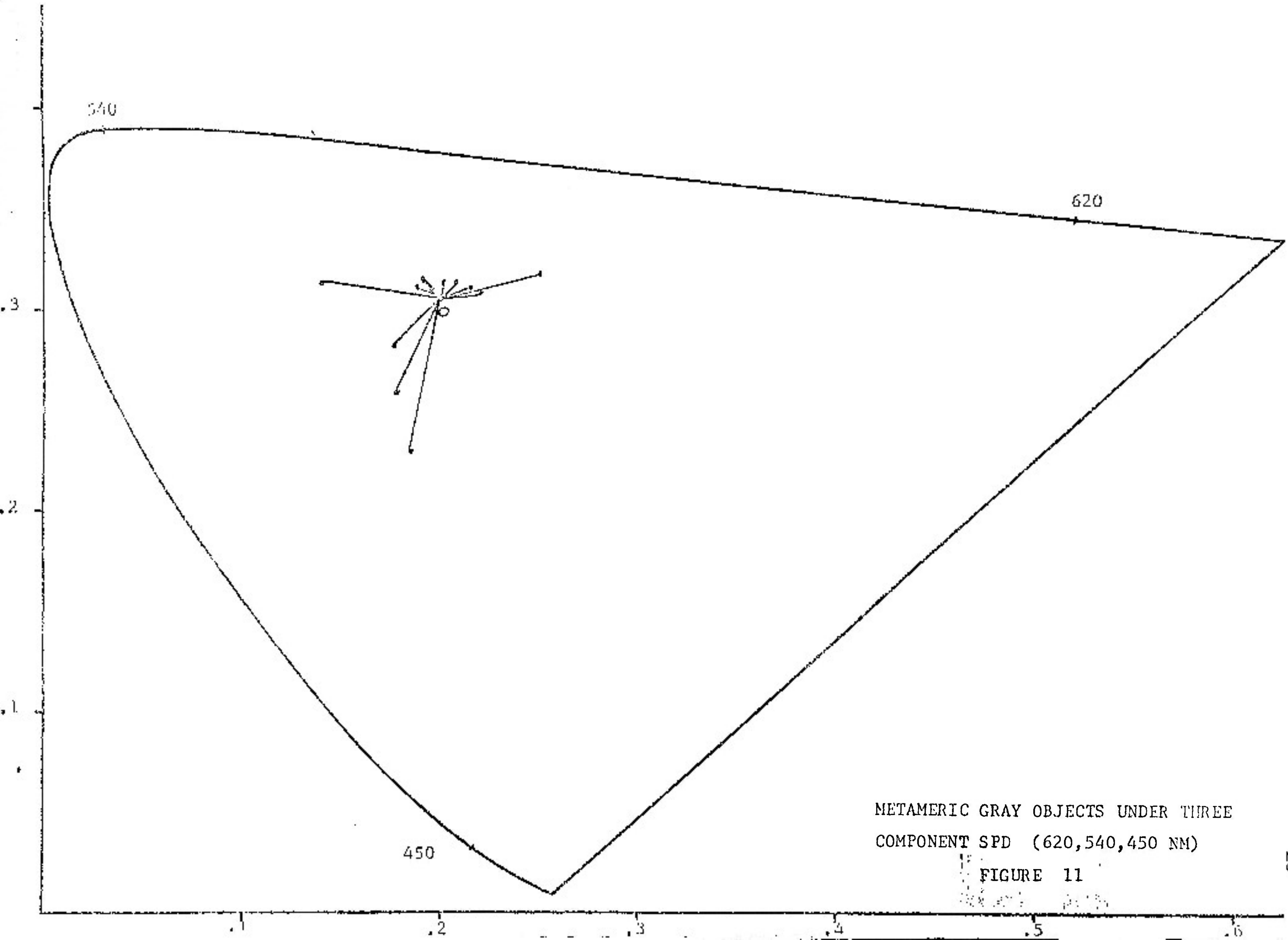
FIGURE 10

at the same wavelengths as Mr. Thornton had postulated. That is the value maximizes at the same point regardless of the choice of test object colors. As the first step the power levels for each of the wavelength combinations were calculated based on the previously given equations and are given in Table 2. These were used as test illuminants in calculation the CRI using the twelve metameric gray objects. The numerical results of these calculations are presented in Table 3. Since the CRI is scaled based on a value of 50 being that of a standard warm white florescent tube and because the color difference of the metameric functions when viewed under the line spectra in this experiment are so great the resulting values of CRI are negative. An illustration of where these object colors fall on the CIE diagram at the maximum point of CRI in the 3^3 factorial experiment is in Figure 11.

In order to determine if the maximum was at the same point the CRI data in Table 3 was placed through a regression analysis program from the Statistical Package for the Social Sciences.[8] The resulting equation was:

$$\begin{aligned} CRI = \; & -22.95 + 7.95C_1 + 13.40C_2 + 5.00C_3 + 10.06C_1^2 - 20.57C_2^2 \\ & - 6.20C_3^2 - 7.54C_1C_2 - 8.94C_2C_3 \end{aligned}$$

Where C_1, C_2, C_3 are the treatment codes for Factors 1 to 3. This equation has an R square maximum of .992 while maintaining a standard error of 2.07. Using this equation and holding C_1 at a constant, lines of constant response at each level of factor C_1 were calculated and plotted in Figures 12 - 14. From these curves and Table 3 it is clear that the CRI value at the treatment combination 0,0,0 or 610, 540, 450 nm is clearly less than the CRI from 620, 540, 450 which is the best CRI under the experimental conditions. If a maximization



METAMERIC GRAY OBJECTS UNDER THREE
COMPONENT SPD (620,540,450 NM)

FIGURE 11

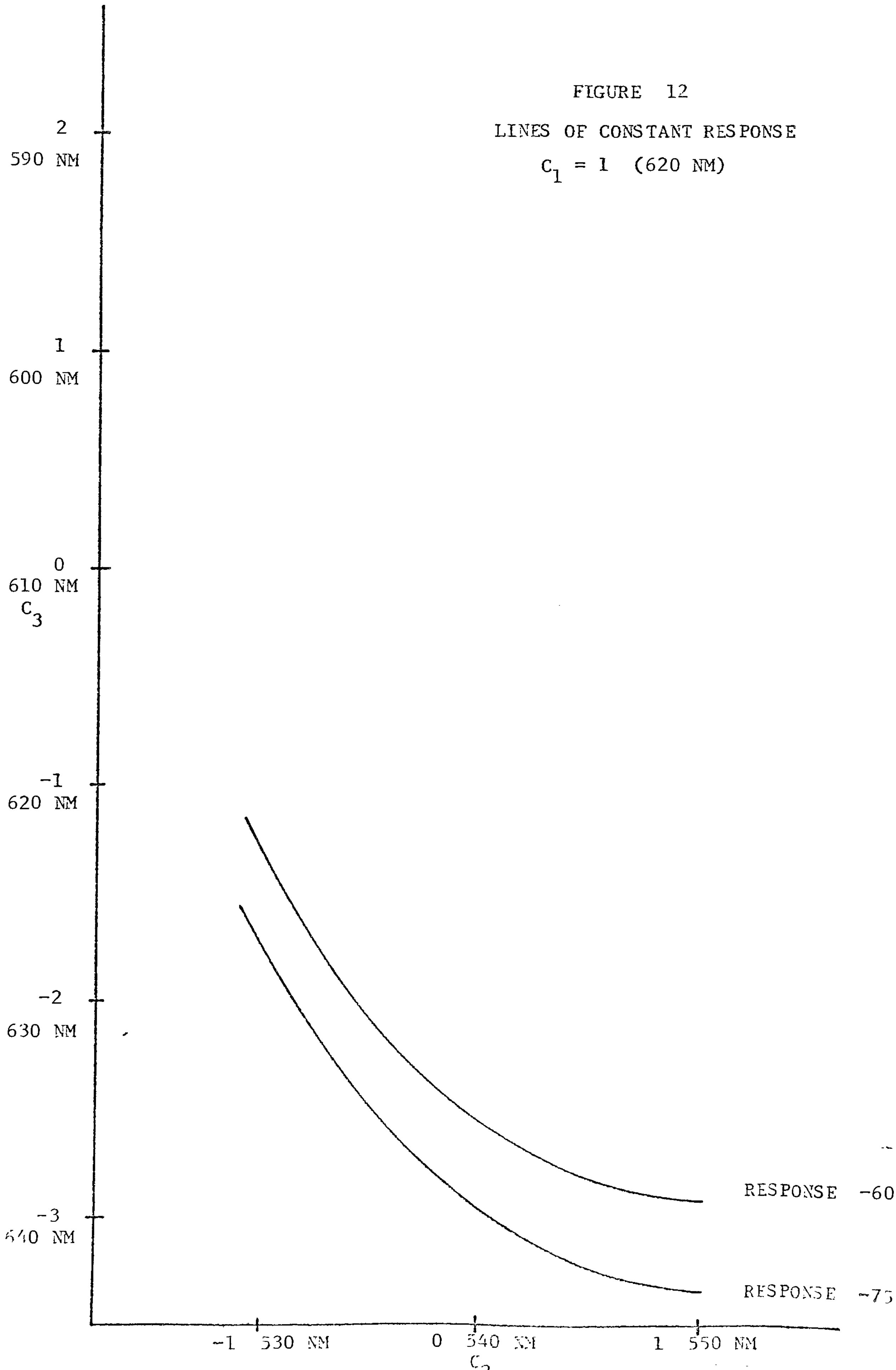


FIGURE 12

LINES OF CONSTANT RESPONSE

$C_1 = 1$ (620 NM)

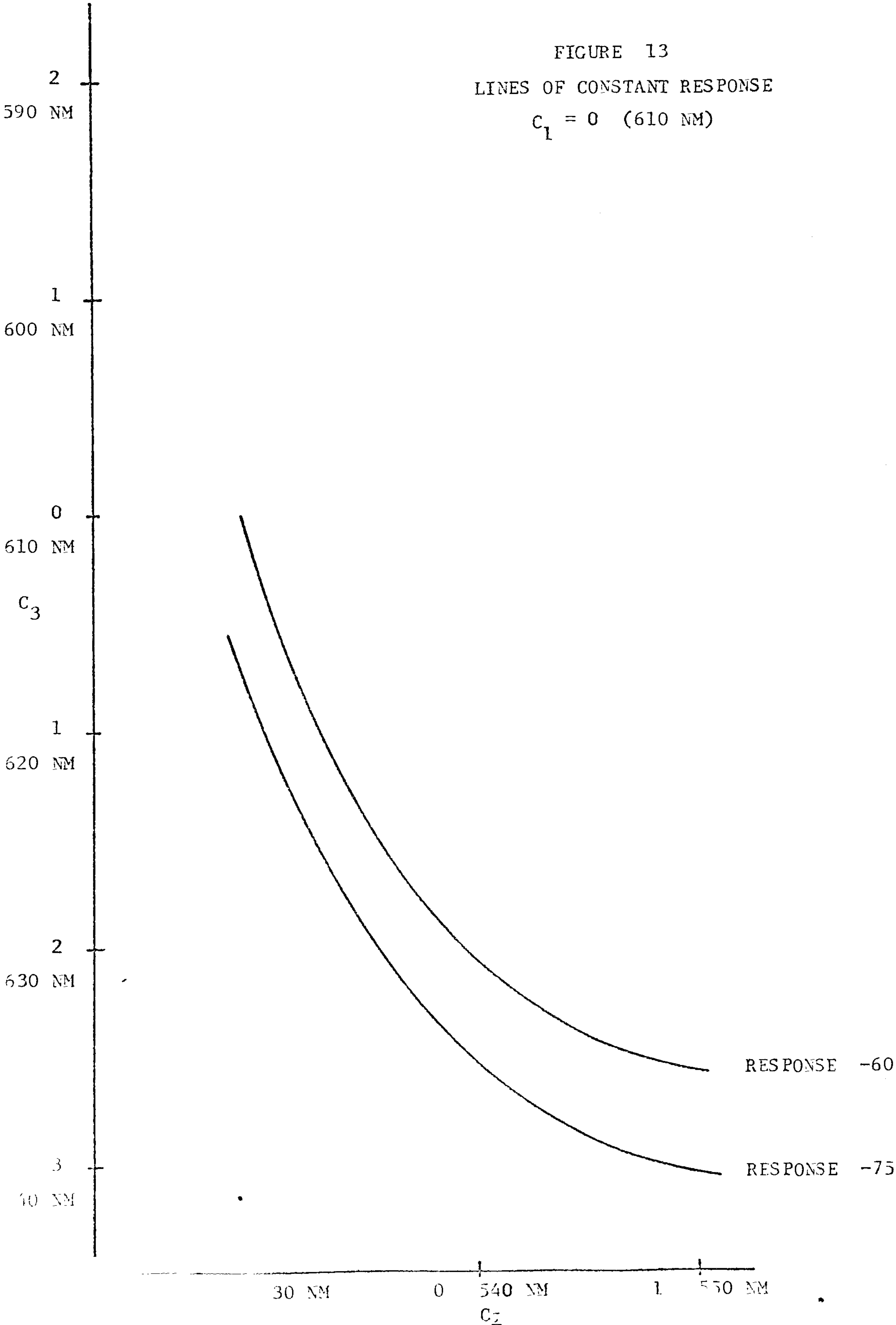


FIGURE 13

LINES OF CONSTANT RESPONSE

$C_1 = 0$ (610 NM)

FIGURE 13

LINES OF CONSTANT RESPONSE

$C_1 = -1$ (600 NM)

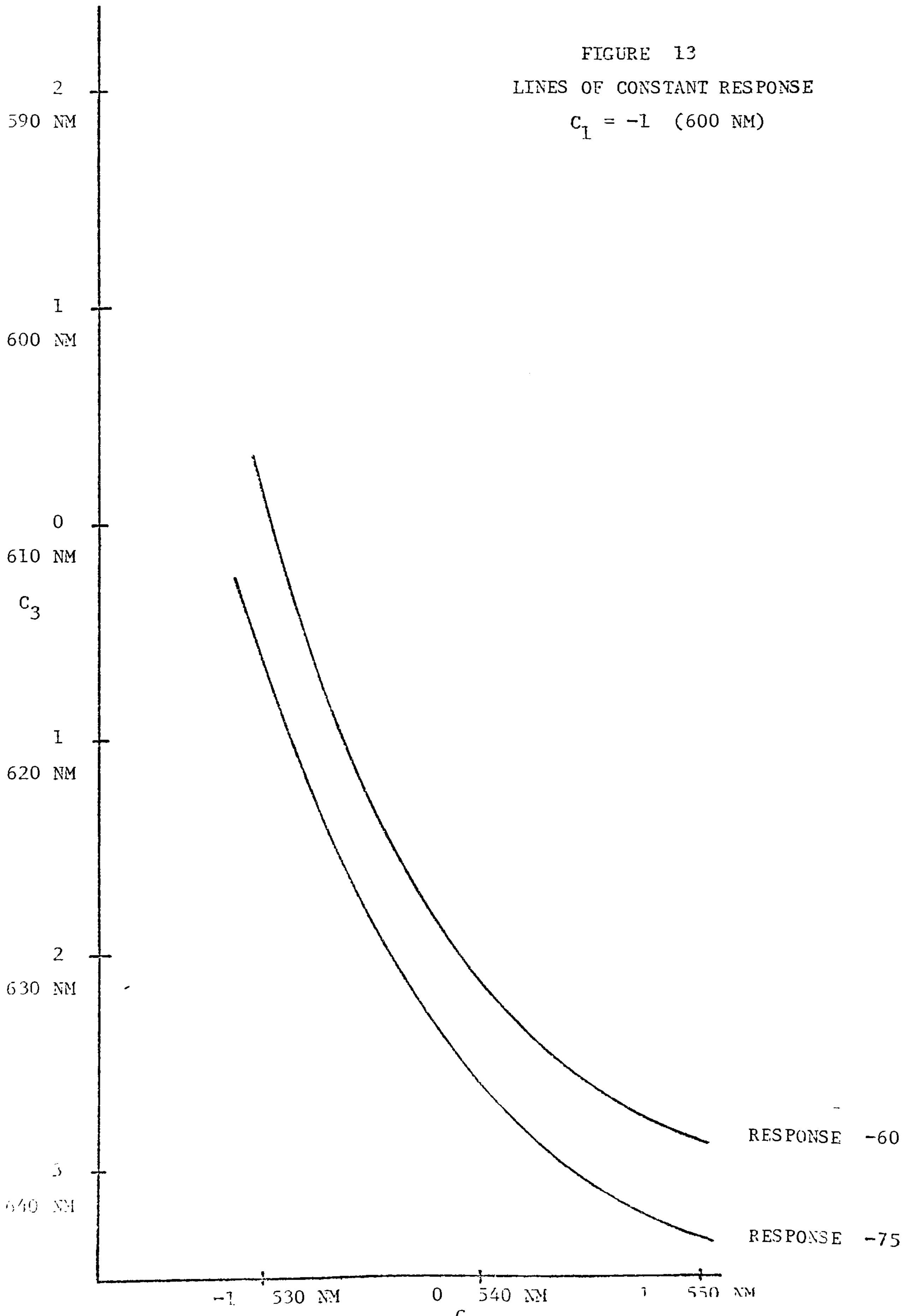


TABLE 2

POWER LEVELS FOR THE 3^3 FACTORIAL EXPERIMENT

TREATMENT COMBINATION	FACTOR 1	LEVEL	FACTOR 2	LEVEL	FACTOR 3	LEVEL
-1,-1,-1	600	59.72	530	70.54	440	65.94
-1,-1, 0	600	61.07	530	68.43	450	65.06
-1,-1, 1	600	63.29	530	64.87	460	69.16
-1, 0,-1	600	51.51	540	69.14	440	66.84
-1, 0, 0	600	53.13	540	67.05	450	65.92
-1, 0, 1	600	55.79	540	63.52	460	70.03
-1, 1,-1	600	40.32	550	73.38	440	67.28
-1, 1, 0	600	42.29	550	71.15	450	66.34
-1, 1, 1	600	45.55	550	67.37	460	70.45
0,-1,-1	610	62.08	530	78.02	440	65.77
0,-1, 0	610	63.49	530	76.09	450	64.89
0,-1, 1	610	65.79	530	72.82	460	68.97
0, 0,-1	610	52.78	540	75.38	440	66.78
0, 0, 0	610	54.44	540	73.49	450	65.86
0, 0, 1	610	57.17	540	70.28	460	69.96
0, 1,-1	610	40.52	550	78.46	440	67.27
0, 1, 0	610	42.51	550	75.48	450	66.33
0, 1, 1	610	45.78	550	73.12	460	70.43
1,-1,-1	620	72.04	530	82.41	440	65.67
1,-1, 0	620	73.67	530	80.59	450	64.78
1,-1, 1	620	76.33	530	77.48	460	68.86
1, 0,-1	620	60.74	540	78.96	440	66.75
1, 0, 0	620	62.64	540	77.18	450	65.82
1, 0, 1	620	65.77	540	74.15	460	69.92
1, 1,-1	620	46.12	550	81.29	440	67.26
1, 1, 0	620	48.38	550	79.45	450	66.32
1, 1, 1	620	52.09	550	76.31	460	70.42

TABLE 3

COLOR RENDERING INDEX DATA FOR METAMERIC GRAY OBJECTS BASED ON TREATMENT COMBINATIONS IN THE 3^3 FACTORIAL EXPERIMENT

TREATMENT COMBINATION	CALCULATED CRI	REGRESSION PREDICTED CRI	RESIDUAL
-1,-1,-1	-81.30	-81.99	.69
-1,-1, 0	-65.50	-62.37	-3.13
-1,-1, 1	-56.60	-55.15	-1.45
-1, 0,-1	-34.20	-31.87	-2.33
-1, 0, 0	-17.70	-20.85	3.15
-1, 0, 1	-19.60	-22.24	2.64
-1, 1,-1	-21.80	-22.89	1.09
-1, 1, 0	-20.30	-20.47	.17
-1, 1, 1	-31.30	-30.47	-.83
0,-1,-1	-73.20	-77.09	3.89
0,-1, 0	-56.90	-56.94	.04
0,-1, 1	-47.30	-49.19	1.89
0, 0,-1	-37.00	-34.17	-2.83
0, 0, 0	-22.90	-22.96	.06
0, 0, 1	-24.40	-24.16	-.24
0, 1,-1	-31.90	-32.39	.49
0, 1, 0	-31.50	-30.13	-1.37
0, 1, 1	-42.20	-40.27	-1.93
1,-1,-1	-52.40	-52.07	-.33
1,-1, 0	-32.60	-31.38	-1.22
1,-1, 1	-23.50	-23.11	-.39
1, 0,-1	-17.60	-16.35	-1.25
1, 0, 0	-3.70	-4.95	1.25
1, 0, 1	-6.40	-5.96	-.44
1, 1,-1	-21.20	-21.78	.58
1, 1, 0	-18.60	-19.66	1.06
1, 1, 1	-29.20	-29.94	.74

of CRI exists for a three line spectra and metameric gray object functions it is not within the bounds of this experiment and in any case it is not at the point of maximization when using the standard CIE/CRI reflectance functions. If the point of maximization were the same the lines of constant response would have a point at which the first derrivative would be zero which would be inside the boundaries of the experiment and clearly this is not the case.

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APPENDIX A

PRINCIPLE COMPONENT ANALYSIS

In photographic science one is often involved in analysis of multivariate response data which could include such things as families of D Log E curves, modulation transfer functions, reflectances etc. As the amount of data collected increases, the ease of comparison and manipulation decreases; thus it is often desireable to reduce the data to a more manageable result. One method ideally suited for such work is characteristic vector analysis sometimes called principle component analysis. The Application of Characteristic Vector Analysis to Photographic and Optical Response Data by J. L. Simonds in the Journal of the Optical Society of America; August 1963; outlines an algorithm for this type of analysis. The computer program is based on this algorithm. A detailed description is provided in Simonds's article; however a short numerical example is helpful in understanding program operations.

Define a matrix of 3 cases (rows) by 6 variables (columns) as:

$$E_s = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 1 & 9 & 8 & 3 & 2 & 4 \\ 3 & 7 & 5 & 8 & 4 & 1 \\ 2 & 5 & 8 & 7 & 3 & 1 \end{bmatrix}$$

The variable (column) average is :

$$\bar{E} = \begin{bmatrix} 2 & 7 & 7 & 6 & 3 & 2 \end{bmatrix}$$

Subtracting $\overline{E}$ from E_s:

$$P = E_s - \overline{E} = \begin{bmatrix} -1 & 2 & 1 & -3 & -1 & 2 \\ 1 & 0 & -2 & 2 & 1 & -1 \\ 0 & -2 & 1 & 1 & 0 & -1 \end{bmatrix}$$

Calculating the variance- covariance matrix:

$$S = P\,P' = \begin{bmatrix} 2 & -2 & -3 & 5 & 2 & -3 \\ -2 & 8 & 0 & 0 & -2 & 6 \\ -3 & 0 & 6 & -6 & -3 & 3 \\ 5 & -8 & -6 & 14 & 5 & -9 \\ 2 & -2 & -3 & 5 & 2 & -3 \\ -3 & 6 & 3 & -9 & -3 & 6 \end{bmatrix}$$

The trace of S:

$$\text{tr } S = \sum_{i=1}^{6} S_{i,i} = 38$$

Arbitrarily defining the first estimate of the characteristic vector:

$$y_0 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Calculating the first itteration x_1:

$$x_1 = S \times y_0 = \begin{bmatrix} 1 & 2 & -3 & 1 & 1 & 0 \end{bmatrix}$$

$$x_1(\max) = 2$$

Normalizing x_1:

$$y_1 = \frac{x_1}{x_1(\max)} = \begin{bmatrix} .5 & 1 & -1.5 & .5 & 0 \end{bmatrix}$$

Continuing the itteration:

.

.

$$x_{11} = \begin{bmatrix} 10.7625 & -18.0152 & -12.5173 & 30.5325 & 10.7625 & -19.7701 \end{bmatrix}$$

$$y_{11} = \begin{bmatrix} .3525 & -.5900 & -.4100 & 1.0000 & .3525 & -.6475 \end{bmatrix}$$

$$x_{12} = \begin{bmatrix} 10.7625 & -18.0152 & -12.5173 & 30.5325 & 10.7625 & -19.7701 \end{bmatrix}$$

At this point x_{11} and x_{12} are equal and the first root is:

$$\lambda_1 = 30.5325$$

The percentage of trace accounted for by the first root is:

$$R = \frac{\lambda_1}{tr\ S} \times 100. = \frac{30.5325}{38} \times 100. = 80.35\ \%$$

Setting the sum of squares of the elements of x_{12} equal to λ_1 the first characteristic vector is obtained:

$$V^1 = \begin{bmatrix} 1.3180 & -2.2061 & -1.5329 & 3.7390 & 1.3180 & -2.4210 \end{bmatrix}$$

The variance described by the first characteristic vector V^1 is removed from the variance-covariance matrix:

$$S_2 = S - V^1 V^{1'}$$

In determining the second characteristic vector V^2 the variance-covariance matrix S_2 is used and after three itterations:

$$x_3 = \begin{bmatrix} 2.1634 & 7.4674 & -8.0605 & .5931 & 2.1634 & 1.5703 \end{bmatrix}$$

$$\lambda_2 = 7.4674$$

The total trace percentage is:

$$R = \frac{\lambda_1 + \lambda_1}{tr\ S} \times 100. = 99.9999\ \%$$

Calculating the second characteristic vector V^2 as before:

$$V^2 = \begin{bmatrix} .5128 & 1.7700 & -1.9106 & .1406 & .5128 & .3722 \end{bmatrix}$$

The weighting functions W are determined by:

$$W^i = \frac{V^{i'}}{\lambda_i}$$

for the example

$$W^1 = \begin{bmatrix} .0432 & -.0723 & -.0502 & .1225 & .4317 & -.0793 \end{bmatrix}$$

$$W^2 = \begin{bmatrix} .0687 & .2370 & -.2558 & .0188 & .0687 & .4984 \end{bmatrix}$$

The scalar multipliers are calculated from the original data:

$$Y_1 = \sum_{i=1}^{r} W_{1,i}(P_{n,i}) \qquad\qquad Y_2 = \sum_{i=1}^{r} W_{2,i}(P_{n,i})$$

where n is the row or case for which the scalars are desired and r is the number of variables in each set of data or function.

Thus to reconstruct case 2 of the original data the scalars are:

$$Y_1 = .5109 \qquad\qquad Y_2 = .6369$$

therefore $E_x = \sum_{i=1}^{nv} (V^i \times Y_i) + \overline{E}$

where nv is the number of characteristic vectors, thus

$$E_2 = \begin{bmatrix} 3 & 7 & 5 & 8 & 4 & 1 \end{bmatrix}$$

In this simple data set the resulting reconstruction is equal to the input function since we have accounted for all the variance. In this simple case principle component analysis is of little value; however its use can be appreciated if in the analysis of a 100 square matrix it would be possible to reduce that data to perhaps three vectors and a set of scalar multipliers.

Program useage

The following description provides an outline of both the job control and data cards required to run program COMPONAL2 on the RIT computer system.

1. JOB statement

   !JOB acct#,acctname(UCC),7.

2. LIMIT statement

   !LIMIT (CORE,32),(TIME,4),(UO,50),(RERUN)

   TIME and UO will vary depending on the amount of data.

3. ASSIGN statement

This program provides the capability to write the input data mean, characteristic vectors and reconstructed functions to a permànant disc file with the format 10F10.4. If this option is selected by any of the problems in the job stream the following cards must be provided; otherwise they are not required.

```
!ASSIGN F:1,(FILE,filename1),(OUT),(SAVE)
!ASSIGN F:2,(FILE,filename2),(OUT),(SAVE)
```

Upon termination of the job filename1 will contain, sequentially, the average of columns, and the characteristic vectors 1 - n. Similarly filename2 will contain the reconstructions of the input functions.

NOTE: Should the job contain more than one problem which selected the write file option the data would be placed in the files in the order in which the problems were run in the job stream.

4. RUN statement

```
!RUN (LMN,COMPANAL2,acct#)
```

where acct# is optional and required only if the program is not in the account in which the job is running.

5. Data Input

A. PROBLM card

Column	Data
1-6	PROBLM
7-9	Number of variables per function ($2 \leq NV \leq 100$)
10-12	Number of cases ($3 \leq NC \leq 50$)
13-15	YES if file write option is desired; otherwise blank.

16-18	YES if printing of reconstructions are desired; otherwise blank.
19-21	Number of variables to be labeled, if desired LABELS cards must be provided. (≤NV)
22,23	Number of variable format cards <10
24-30	Fraction of trace to be accounted for, F7.5 format (.01≤PCTN≤1.00) if blank default is for .99 or 99 %.

B. TITLE card

Column	Data
1-5	TITLE
6	blank
7-78	problem title

C. LABELS card (if required)

Column	Data
1-6	LABELS
7-...	Format 7(I4,A6) each variable to be labeled must have its number and associated title punched into a LABELS card. If more than 7 variables are to be labeled then repeat this step until completed.

D. Variable format card

The format card will vary depending on the number of variables in each function and how they are punched into the data cards. As example if the number of variables were 95 (NV) and each card was punched with 20 values punched in 20F4.3 format the card would be:

(20F4.3/20F4.3/20F4.3/20F4.3/15F4.3)

The '/' being required to skip to the next card resulting in a total of 95 values being read.

E. Data

Punched according to the variable format card specified in D.

Each case must be on a seperate set of cards.

(Repeat A - E as required for each problem in the job stream.)

F. FINISH card

Column	Data
1-6	FINISH

6. EOD card

!EOD

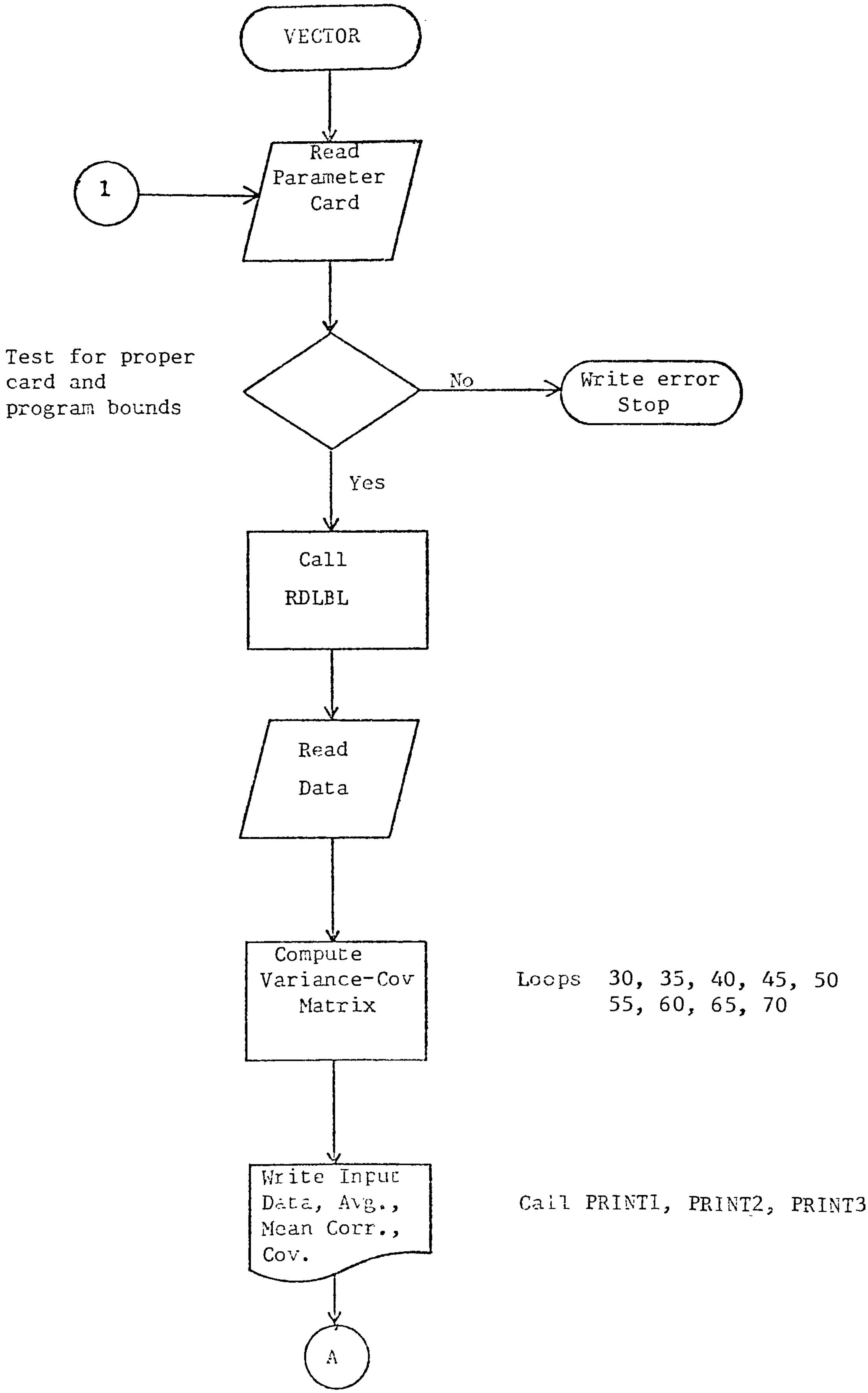
VECTOR
1
Read Parameter Card
Test for proper card and program bounds
No
Write error Stop
Yes
Call RDLBL
Read Data
Compute Variance-Cov Matrix
Loops 30, 35, 40, 45, 50 55, 60, 65, 70
Write Input Data, Avg., Mean Corr., Cov.
Call PRINT1, PRINT2, PRINT3
A

FIGURE A-1

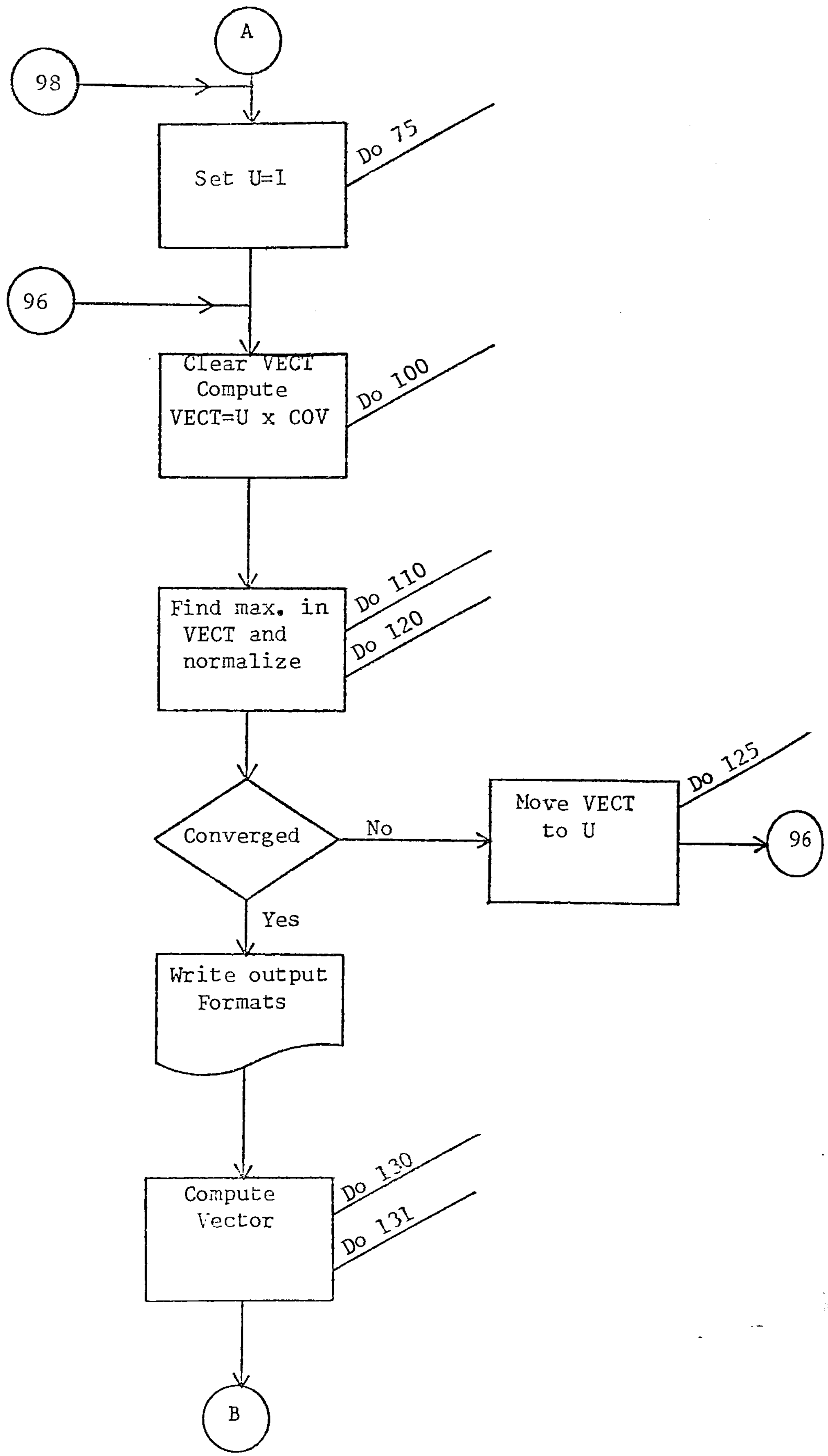

FIGURE A-1 (CONTINUED)

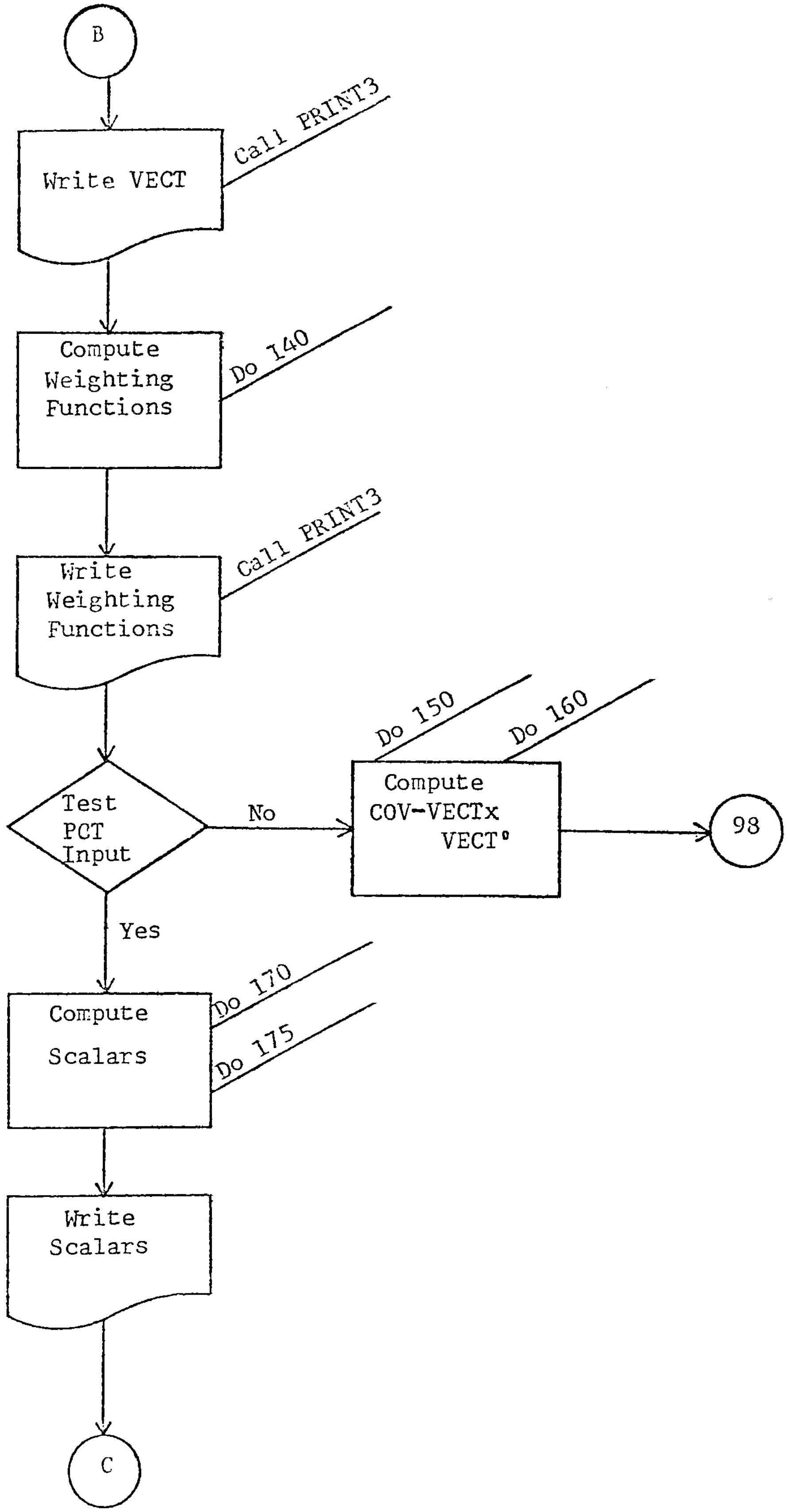

FIGURE A-1 (CONTINUED)

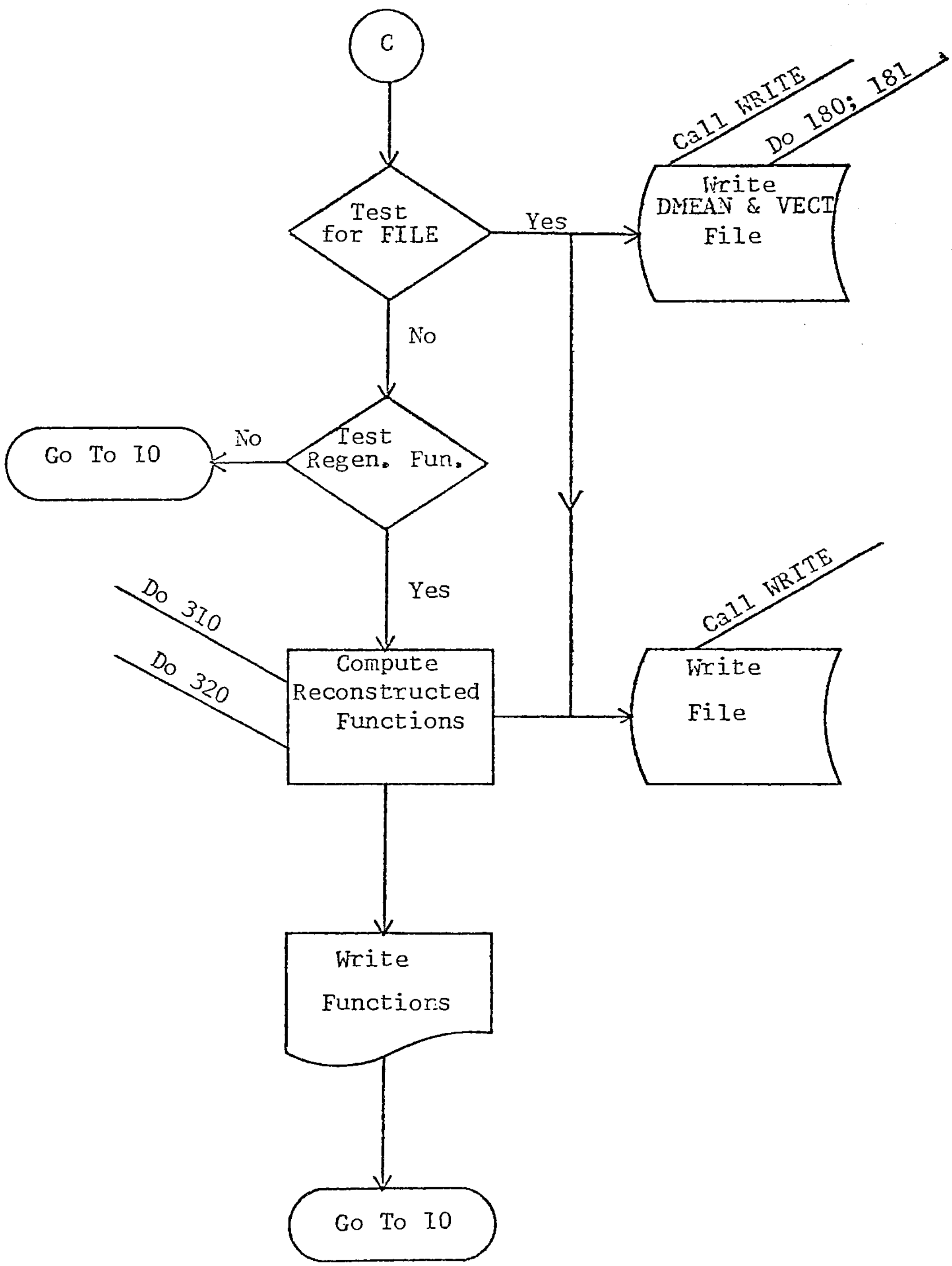

FIGURE A-1 (CONTINUED)

SUBROUTINE WRITE

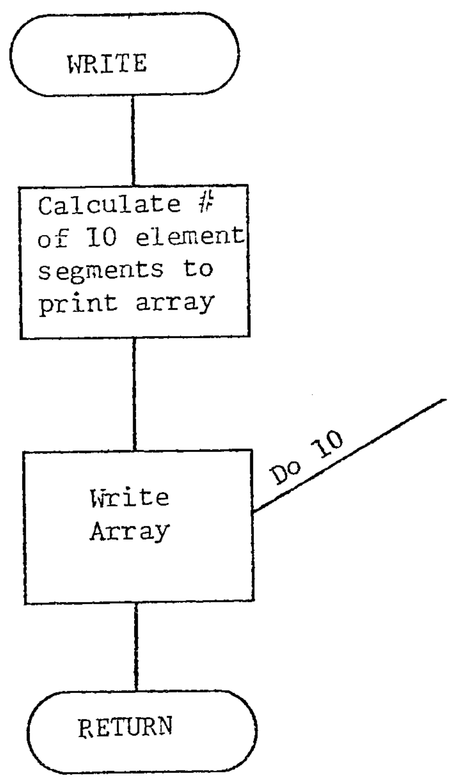

FIGURE A-I (CONTINUED)

SUBROUTINE RDLBL

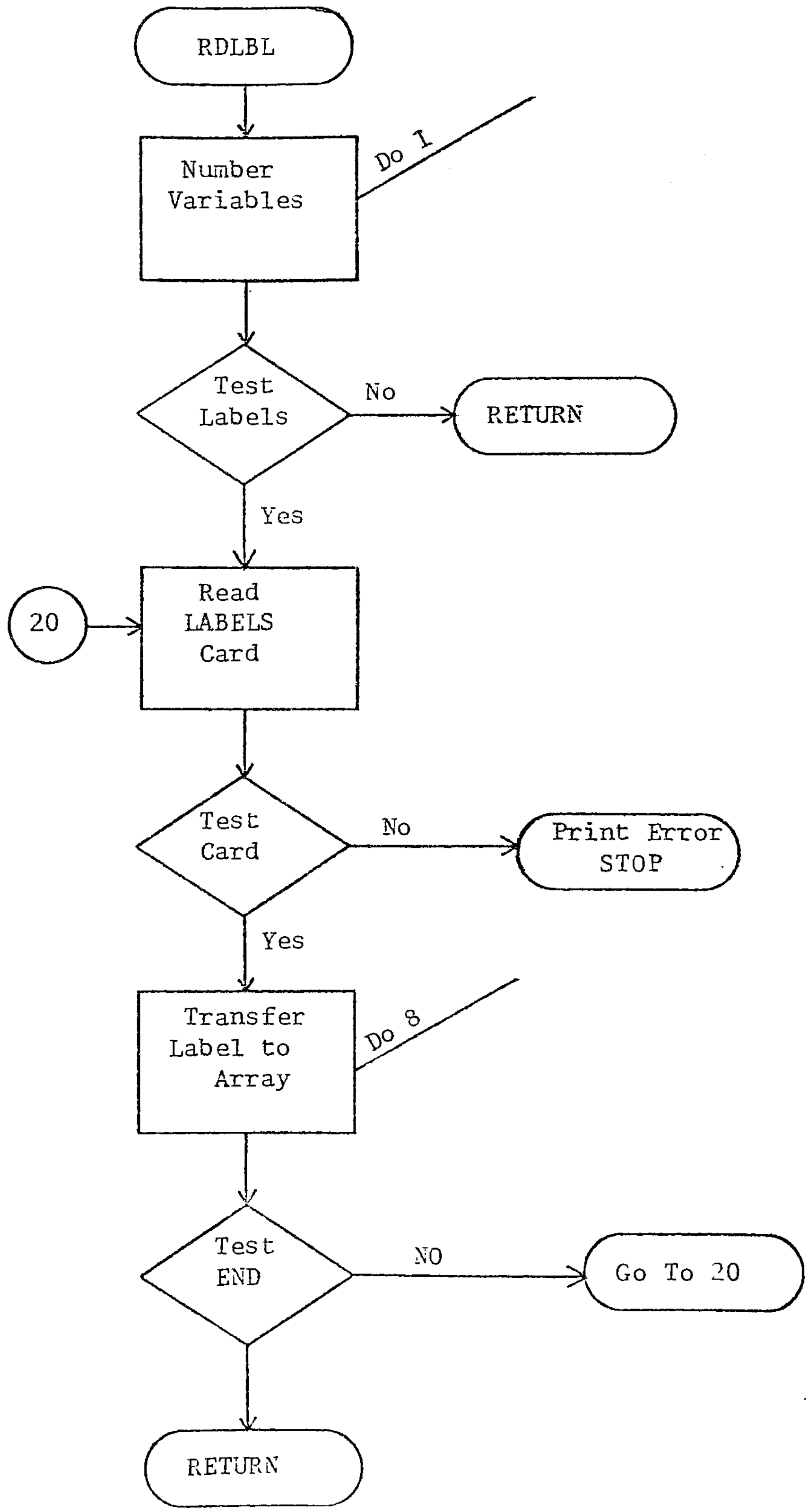

FIGURE A-1 (CONTINUED)

SUBROUTINE PRINTI

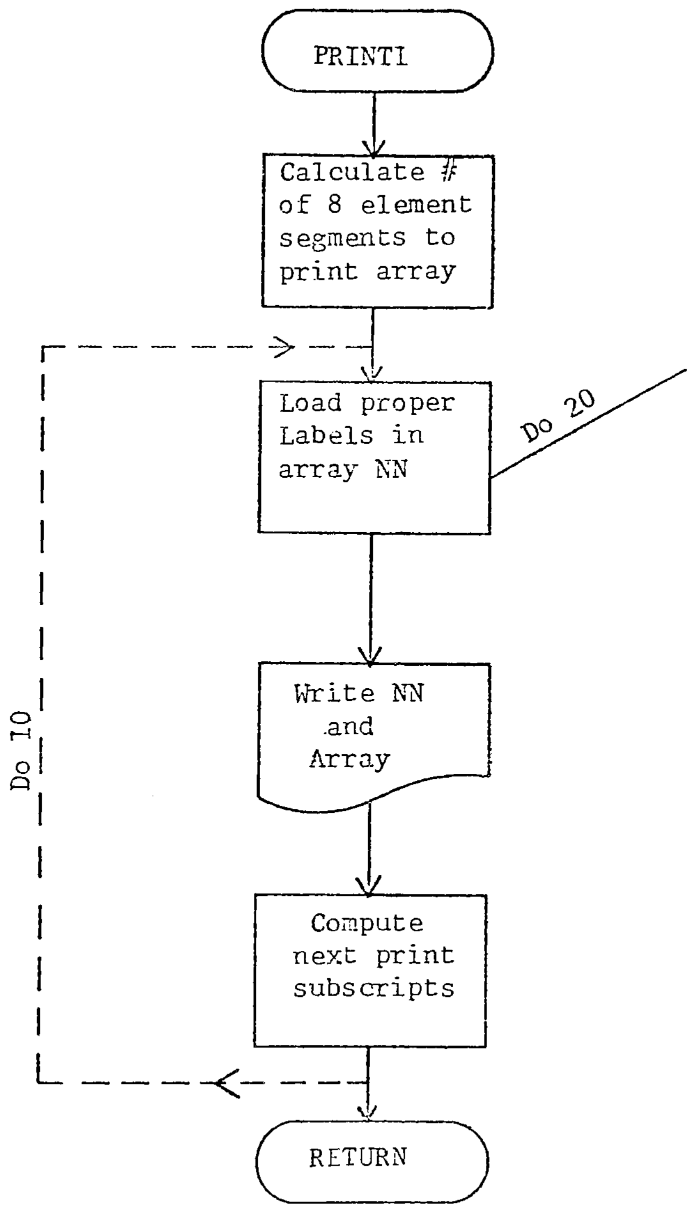

FIGURE A-1 (CONTINUED)

SUBROUTINE PRINT2

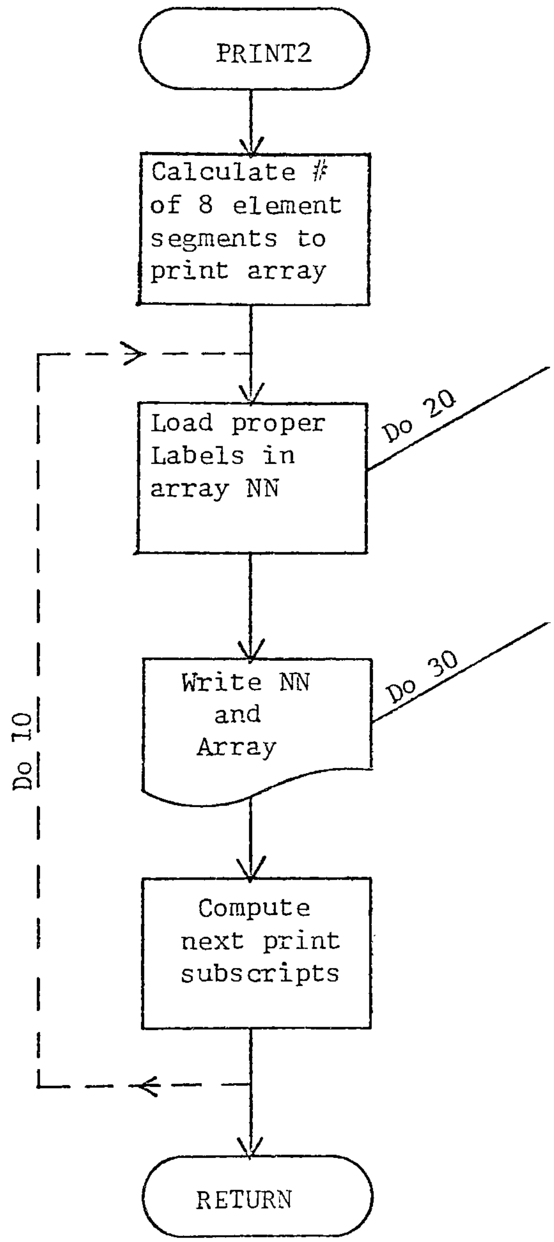

FIGURE A-1 (CONTINUED)

SUBROUTINE PRINT3

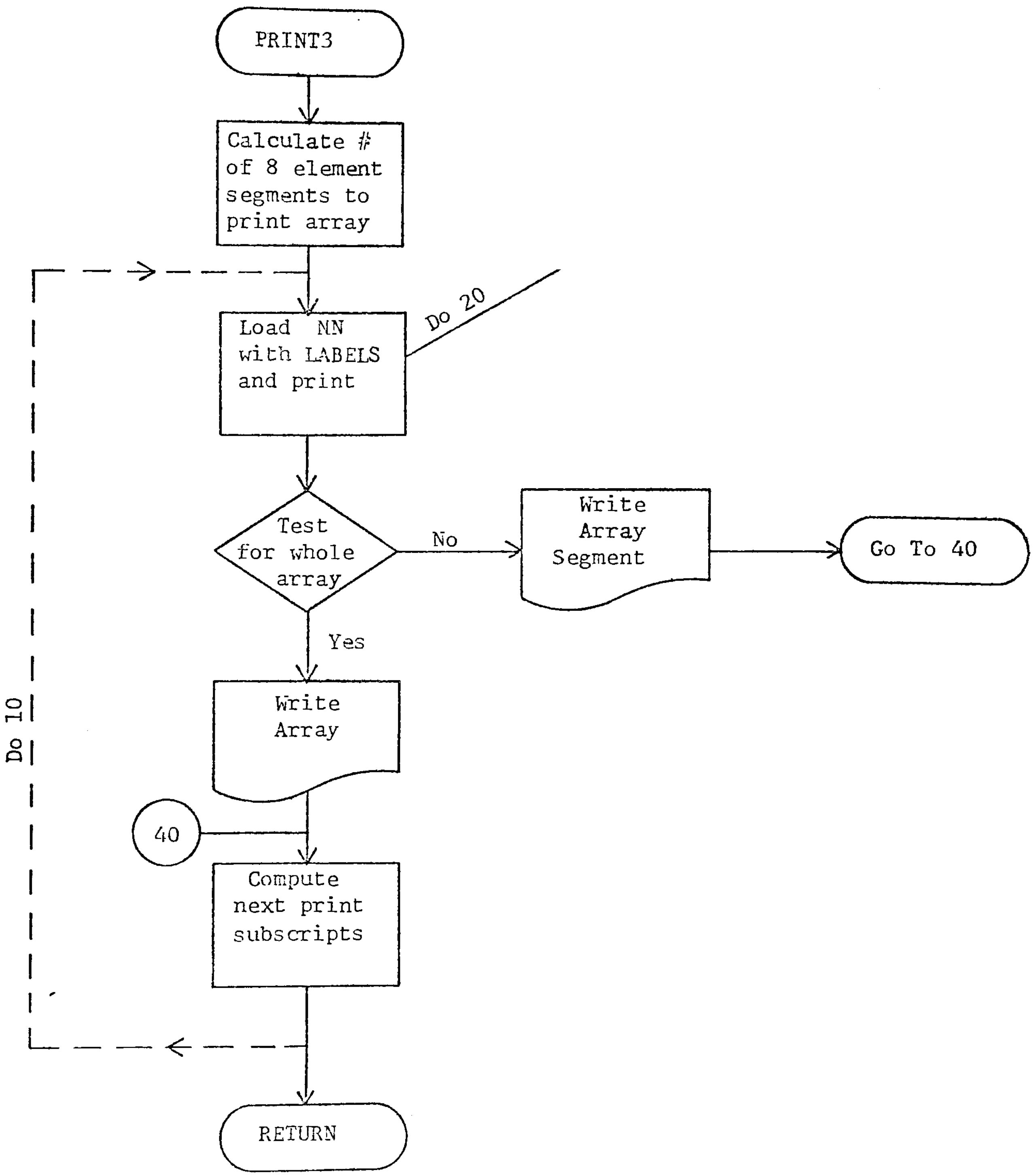

FIGURE A-1 (CONTINUED)

PRINCIPLE COMPONENT ANALYSIS

COMPUTER PROGRAM

FIGURE A-2

```
C
C     PRINCIPLE COMPONENT ANALYSIS
C     FOR BACKGROUND SEE
C        'APPLICATION OF CHARACTERISTIC VECTOR ANALYSIS TO PHOTOGRAPHIC
C          AND OPTICAL RESPONSE DATA'  J. L. SIMONDS, JOURNAL OF THE
C          OPTICAL SOCIETY OF AMERICA, VOL. 53, AUGUST 1953, P. 968-974
C
C     D IS MATRIX FOR INPUT DATA
C     DMEAN IS ARRAY OF COLUMN AVERAGES OF MATRIX D
C     COV IS ARRAY FOR VARIANCE-COVARIANCE MATRIX
C     VECT IS ARRAY FOR CHARACTERISTIC VECTORS
C     FMT IS ARRAY FOR VARIABLE FORMAT CARDS
C     LABELS IS ARRAY OF LABELS FOR INPUT DATA
C     SCALAR IS ARRAY FOR SCALAR MULTIPLES
C     TITLE IS ARRAY FOR PROBLEM SET TITLE
C     U IS COMPUTATIONAL ARRAY
C     W IS ARRAY OF WEIGHTING FUNCTIONS
C
C*************************************************************************
C     DUE TO SIZE RESTRICTIONS, THIS PROGRAM IN ITS PRESENT VERSION
C         CAN CALCULATE ONLY 50 CHARACTERISTIC VECTORS BASED ON THE
C         INPUT DATA.  IN ORDER TO INCREASE THE PROGRAM CAPABILITY
C         THE DIMENSIONS OF VECT, SCALAR, AND W WOULD HAVE TO BE
C         INCREASED TO 100.  THE APPROPRIATE CHANGES IN SUBROUTINES
C         PRINT2 AND PRINT3 WOULD HAVE TO BE MADE ALSO.
C*************************************************************************
C
      COMMON /IO/LRD,LPU,LPR
      REAL*8 A1,B1,CODE,FINISH,PROBLM,LABELS,TITLE ,C1
      DATA A1,B1,C1,D1,E1/'FINISH','PROBLM','TITLE ',' YES',' NO'/
      DIMENSION D(50,100),DMEAN(100),COV(100,100),VECT(50,100),FMT(120),
     1          LABELS(100),SCALAR(50,50) ,TITLE(12),U(100),W(50,100)
      COMMON D,COV,VECT,SCALAR,W
C     IO DEVICE ASSIGNMENTS
      LRD=105
      LPU=106
      LPR=108
   10 READ (LRD,900) CODE,NV,NC,PALL,PALL1,LBLV,FCD,PCTN
      IF (A1 .EQ. CODE) GO TO 15
      GO TO 20
   12 WRITE (LPR,901) CODE
   15 WRITE (LPR,902)
      STOP
   20 IF (B1 .NE. CODE) GO TO 12
      IF (PALL .NE. D1) PALL=E1
      IF (PALL1 .NE. D1) PALL1=E1
      IF (PCTN) 5,5,7
    5 PCTN=.97
    7 READ (LRD,903) CODE, (TITLE(I),I=1,12)
      IF (C1 .NE. CODE) GO TO 12
C     TEST FOR PROBLEM BOUNDS
      IF (PALL .EQ. D1) PALL1=D1
      IF ((PCTN-.01)*(PCTN-1.001)) 19,202,202
   19 IF ((NV-1)*(NV-101)) 21,200,200
   21 IF ((NC-2)*(NC-51)) 22,201,201
   22 IF (FCD .GT. 0 .AND. FCD .LE. 10) GO TO 24
      FCD=1
      WRITE (LPR,904)
   24 WRITE (LPR,905) (TITLE(I),I=1,12),NV,NC,FCD,PALL,PALL1,PCTN*100.0
      CALL RDLBL(LBLV,NV,LABELS)
      FCD=FCD*12
      READ (LRD,906) (FMT(I),I=1,FCD)
      WRITE (LPR,907) (FMT(I),I=1,FCD)
      DO 25 I=1,NC
   25 READ (LRD,FMT) (D(I,J),J=1,NV)
      WRITE (LPR,908)
      TESTA=0.0
      CALL PRINT3(D,NV,NC,LABELS,0)
C     FIND AVERAGE D
      DO 30 J=1,100
   30 DMEAN(J)=0.0
      DO 35 I=1,NV
      DO 40 J=1,NC
   40 DMEAN(I)=DMEAN(I)+D(J,I)
      DMEAN(I)=DMEAN(I)/NC
   35 CONTINUE
      WRITE (LPR,909)
      CALL PRINT1(DMEAN,NV,LABELS)
C     SUBTRACT AVERAGE FROM D
      DO 45 I=1,NC
```

```
   50 D(I,J)=D(I,J)-UMEAN(J)
   45 CONTINUE
C     WRITE MEAN CORRECTED 'D' MATRIX
      WRITE (LPR,914)
      CALL PRINT3(D,NV,NC,LABELS,0)
C     CALCULATE D*TRANSPOSE*D AND ITS TRACE
      TRACE=0.0
      DO 55 J=1,NV
      DO 60 I=1,NV
      COV(J,I)=0.0
      DO 65 K=1,NC
   65 COV(J,I)=COV(J,I)+D(K,J)*D(K,I)
      IF (I-J) 60,70,60
   70 TRACE=TRACE+COV(J,I)
   60 CONTINUE
   55 CONTINUE
C     WRITE MATRIX AND TRACE
      WRITE (LPR,910)
      CALL PRINT2(COV,NV,LABELS)
      WRITE (LPR,911) TRACE
      NVECT=0
   93 AB=0.0
      NVECT=NVECT+1
C     TEST FOR .GT. 50 CHARACTERISTIC VECTORS
      IF (NVECT .GT. 50) GO TO 902
C     SET U ARRAY TO 1
      DO 75 I=1,NV
   75 U(I)=1.0
C     COUNT IS NUMBER OF ITERATIONS/EIGENVALUE
      COUNT=0.0
C     COMP IS TEST FOR CONVERGENCE
C     PRODUCT OF UNIT ROW VECTOR AND COV MATRIX
C     CLEAR VECT
   96 DO 100 I=1,NV
      VECT(NVECT,I)=0.0
C     MULTIPLY U BY COV AND STORE IN VECT
      DO 100 J=1,NV
  100 VECT(NVECT,I)=VECT(NVECT,I)+U(J)*COV(I,J)
      COUNT=COUNT+1.0
C     FIND MAXIMUM OF VECTOR, SL
  106 SL=0.0
      DO 110 I=1,NV
      IF (VECT(NVECT,I)-SL) 110,110,115
  115 SL=VECT(NVECT,I)
  110 CONTINUE
      COMP=0.0
C     NORMALIZE VECTOR
      DO 120 I=1,NV
  120 VECT(NVECT,I)=VECT(NVECT,I)/SL
  123 COMP=SL-AB
      AB=SL
C     COMP-TEST IS CONVERGENCE CRITERION
      TEST=AB*0.000001
      COMP=ABS(COMP)
      IF (COMP-TEST) 99,99,121
C     DID NOT CONVERGE    MOVE VECT(NVECT) TO U
  121 DO 125 I=1,NV
  125 U(I)=VECT(NVECT,I)
      GO TO 96
C     CONVERGED, WRITE EIGENVALUE AND NUMBER OF ITTERATIONS NEEDED
   99 WRITE (LPR,915) NVECT
      WRITE (LPR,916) SL
      WRITE (LPR,917) COUNT
C     TESTA IS SUMATION OF EIGENVALUES
C     PCT IS PERCENTAGE OF TRACE EXPLAINED BY EIGENVALUES
      TESTA=TESTA+SL
      PCT=TESTA/TRACE
      WRITE (LPR,918) PCT
C     COMPUTE CHARACTERISTIC VECTOR
      SUMSQ=0.0
      DO 130 I=1,NV
  130 SUMSQ=SUMSQ+VECT(NVECT,I)**2
      CUML=SQRT(SL/SUMSQ)
      DO 131 I=1,NV
  131 VECT(NVECT,I)=VECT(NVECT,I)*CUML
      WRITE (LPR,919)
      CALL PRINT3(VECT,NV,NC,LABELS,NVECT)
      DO 140 I=1,NV
  14[illegible] [illegible](NVECT,I)=VECT(NVECT,I)/SL
      WRITE (LPR,9[illegible])
      CALL PRINT3([illegible],NV,NC,LABELS,NVECT)
```

```
      IF (PCT-PCTN) 159,169,169
C     IF LESS THAN 99 PERCENT, COMPUTE NEXT CHARACTERISTIC VECTOR.
C     COMPUTE COV=VECT(NVECT) TRANSPOSE*VECT(NVECT)
  159 DO 150 I=1,NV
      DO 150 J=1,NV
  150 COV(I,J)=COV(I,J)-VECT(NVECT,I)*VECT(NVECT,J)
  150 CONTINUE
  151 GO TO 95
C     MORE THAN 99 PERCENT EXPLAINED BY EIGENVALUES
C     COMPUTE Y SCALARS
  169 WRITE (LPR,952)
      DO 170 I=1,NVECT
      WRITE (LPR,920) I
      DO 175 K=1,NC
      SCALAR(I,K)=0.0
      DO 175 J=1,NV
  175 SCALAR(I,K)=SCALAR(I,K)+W(I,J)*D(K,J)
      WRITE (LPR,921) K, SCALAR(I,K)
  170 CONTINUE
C     PRINT RECONSTRUCTED AND DELTA DATA IF PALL1 .EQ. YES
C     CREATE RECONSTRUCTED DATA FROM VECTORS
      IF (PALL1 .NE. D1) GO TO 303
      PCT=PCT*100.0
      WRITE (LPR,954) NVECT,PCT
C     WRITE VECTOR FILE IF PALL .EQ. YES
      IF (PALL .NE. D1) GO TO 299
      CALL WRITE(1,DMEAN,NV,ILOOP)
      DO 180 IFLE=1,NVECT
      K11=1
      K22=MIN0(10,NV)
      DO 181 JXFL=1,ILOOP
      WRITE (1,990) (VECT(IFLE,JFLE),JFLE=K11,K22)
      K11=K22+1
      K22=K11+9
      K22=MIN0(K22,NV)
  181 CONTINUE
  990 FORMAT (10F10.4)
  180 CONTINUE
  299 DO 300 I=1,NC
C     CLEAR U ARRAY
      DO 305 I1=1,NV
  305 U(I1)=0.0
      DO 310 J=1,NVECT
      DO 320 K=1,NV
  320 U(K)=U(K)+SCALAR(J,I)*VECT(J,K)
  310 CONTINUE
      DO 322 I2=1,NV
  322 U(I2)=U(I2)+DMEAN(I2)
      WRITE (LPR,935) I
      CALL PRINT1(U,NV,LABELS)
C     WRITE RECONSTRUCTED FUNCTION FILE IF PALL .EQ. YES
      IF (PALL .NE. D1) GO TO 300
      CALL WRITE(2,U,NV,ILOOP)
  300 CONTINUE
      DO 800 I=1,NVECT
      WRITE (LPR,980) I
      DO 810 J=1,NV
      VECTT=VECT(I,J)+DMEAN(J)
      WRITE (LPR,981) LABELS(J),VECT(I,J),DMEAN(J),VECTT
  810 CONTINUE
  800 CONTINUE
  980 FORMAT ('1 VECTOR ',I4,' DATA',//,' POINT',10X,'VECTOR VALUE',
     112X,'DATA MEAN',21X,'VECTOR + DATA MEAN')
  981 FORMAT (2X,A6,9X,E15.6,14X,E15.6,20X,E15.6)
      GO TO 305
  302 WRITE (LPR,940)
  303 GO TO 10
  200 WRITE (LPR,922) NV
      GO TO 15
  201 WRITE (LPR,923) NC
      GO TO 15
  202 WRITE (LPR,960)
      PCTN=.99
      GO TO 15
  900 FORMAT (A6,2I3,2A3,I3,I2,F7.5)
  901 FORMAT (' PROGRAM EXPECTED PROBLEM, TITLE OR FINISH CARD BUT READ'
     1,A6)
  902 FORMAT ('1 FINISH CARD ENCOUNTERED -- PROCESSING ENDED')
  903 FORMAT (13A6)
  904 FORMAT (' NUMBER OF VARIABLE FORMAT CARDS INCORRECTLY SPECIFIED, A
```

```
905 FORMAT ('1 CHARACTERISTIC VECTOR ANALYSIS - VERSION OF JAN 13,76',
   1//,' PROBLEM TITLE: ',12A6,/,' NUMBER OF VARIABLES ',I3,/,
   2' NUMBER OF CASES ',I3,/,' NUMBER OF VARIABLE FORMAT CARDS ',
   3I3,/,' WRITE VECTOR RECONSTRUCTED FUNCTION FILES: ',A3,/,' PRI
   4NT RECONSTRUCTED FUNCTIONS: ',A3,/,' PERCENT OF TRACE TO BE ACCO
   5UNTED FOR ',F7.3,////)
906 FORMAT (18A4)
907 FORMAT (' VARIABLE FORMAT IS',/,(2X,18A4),//)
908 FORMAT ('0 INPUT DATA')
909 FORMAT ('0 AVERAGE OF COLUMNS')
910 FORMAT ('1 VARIANCE-COVARIANCE MATRIX')
911 FORMAT (/////,' TRACE OF VARIANCE-COVARIANCE MATRIX ',E15.6,/////)
914 FORMAT ('1 MEAN CORRECTED MATRIX')
915 FORMAT ('1 VECTOR ',I4,/,'0 EIGENVALUE ')
916 FORMAT (1X,E15.6,/,'0 NO. OF ITTERATIONS')
917 FORMAT (F5.0)
918 FORMAT ('0 SUM EIGENVALUES/TRACE',/,1X,E15.6)
919 FORMAT ('0 CHARACTERISTIC VECTOR')
920 FORMAT ('0 CASE',15X,'Y SCALAR FOR VECTOR ',I3)
921 FORMAT (I7,18X,E15.6)
922 FORMAT (1X,I4,' ORIGINAL VARIABLES IS ILLEGAL')
923 FORMAT (1X,I4,' CASES IS ILLEGAL')
925 FORMAT ('0 WEIGHTING FUNCTIONS')
930 FORMAT (///,' CASE ',I4,/,'0 RECONSTRUCTED DATA')
940 FORMAT ('1 PROGRAM HAS ATTEMPTED TO CALCULATE MORE THAN 50 CHARACT
   1ERISTIC VECTORS',/,' ***** PROCESSING OF THIS PROBLEM DISCONTINUE
   2D *****')
952 FORMAT ('1 SCALAR MULTIPLIERS FOR CALCULATED VECTORS')
954 FORMAT('1 *************** VECTOR RECONSTRUCTIONS ***************
   1*',/,' RECONSTRUCTIONS ARE BASED ON ',I3,' CHARACTERISTIC VECTORS
   2.',/,' THESE VECTORS ACCOUNT FOR ',F7.3,' % OF THE VARIANCE.')
960 FORMAT (2X,I4,'0 PERCENT OF TRACE TO BE ACCOUNTED FOR IS ILLEGAL.
   1 SET TO 99 %.',//)
      END
```

```
      SUBROUTINE WRITE(INO,A,NV,IT)
      DIMENSION A(1)
      IT=1
      KK=0
      K1=1
      K2=MIN0(10,NV)
    1 KK=KK+10
      IF (NV-KK) 5,5,3
    3 IT=IT+1
      GO TO 1
    5 DO 10 JX=1,IT
      WRITE (INO,100) (A(I),I=K1,K2)
      K1=K2+1
      K2=K1+9
      K2=MIN0(K2,NV)
   10 CONTINUE
  100 FORMAT (10F10.4)
      RETURN
      END
```

```
      SUBROUTINE RDLBL(NLBVAR,NVAR,ARRAY)
C
C     SUBROUTINE RDLBL
C     SUBROUTINE TO READ IN LABELS CARDS, STORE THEM IN ARRAY,
C     AND SUBSTUTE NUMBERS FOR UNLABELED VARIABLES
C     NVAR IS TOTAL NUMBER OF VARIABLES
C     NLBVAR IS NUMBER OF LABELED VARIABLES EXPECTED
C     EQUIVALECCE INTEGER AND FLOATING NAMES SO THAT INTEGER SUBTRACTION
C     MAY BE USED TO TEST ALPHABETIC EQUALITY
C
      REAL*8ARRAY,ANUMB,DUMY,TEST,ALABEL
      DIMENSION ARRAY(1),IDUM(7),DUMY(7)
      COMMON /IO/LRD,LPU,LPR
      DATA ALABEL/'LABELS'/
C     NUMBER VARIABLES
      DO 1 I=1,NVAR
    1 ARRAY(I)=ANUMB(I)
C     IF NO LABELS, RETURN
      IF (NLBVAR) 9,9,2
    2 N=0
C     READ 1 LABELS CARD
   20 READ (LRD,3) TEST,(IDUM(J),DUMY(J),J=1,7)
C     TEST FOR 'LAB' IN FIRST THREE COLS.
      IF (TEST-ALABEL) 4,6,4
C     ERROR  PRINT MESSAGE AND QUIT
    4 WRITE (LPR,5) TEST
      STOP
C     EXAMINE 7 FIELDS
    6 DO 8 J=1,7
      K=IDUM(J)
C     TEST INDEX, IF 0, IGNORE,  IF ILLEGAL, PRINT MESSAGE AND
C     IGNORE EXCEPT TO COUNT
      IF(K) 11,8,10
   10 IF (K-NVAR) 7,7,11
   11 WRITE (LPR,12) K,DUMY(J)
      GO TO 13
C     MOVE LABEL TO ARRAY
    7 ARRAY(K)=DUMY(J)
C     STEP NUMBER OF VARIABLES
   13 N=N+1
C     TEST FOR END. IF END, RETURN, IF NOT, SCAN OTHER FIELDS.
      IF (N-NLBVAR) 8,9,9
    8 CONTINUE
      GO TO 20
    9 RETURN
    3 FORMAT (A6,7(I4,A6))
    5 FORMAT (' LABELS CARD EXPECTED BUT READ ',A6)
   12 FORMAT (18H0LABELS CARD INDEX,I7,13H INCORRECT, LABEL ,A6,9H IGNOR
     1ED.)
      END
```

```
      REAL FUNCTION ANUMB*8 (II)
C
C     FUNCTION ANUMB
C
C     THE FUNCTION 'ANUMB' CONVERTS THE INTEGER 'I' TO A RIGHT JUSTIFIED
C     ALPHANUMERIC CHARACTERS WHICH ARE RETURNED AS THE HIGH ORDER FOUR
C     BYTES OF THE REAL*8 VARIABLE 'ANUMB'.
C
      REAL*8 GGGG/'        '/,ALPHA
C
C     THE ARRAY 'IFAKE' IS A MASK CONSISTING OF THE BINARY CHARACTERS
C     '11110000' MINUS THE BINARY CHARACTERS '01000000'.   THE FIRST
C     IS THE MASK WHICH MUST BE ADDED TO AN INTERNAL DIGIT TO MAKE IS
C     ALPHANUMERIC, AND THE SECOND IS THE BINARY CODE FOR A BLANK, WHICH
C     MUST BE SUBTRACTED SINCE IALPH IS INITIALIZED AT BLANKS.
C
      DIMENSION IFAKE(4)
C
C     THE FOLLOWING DATA-INITIALIZATION CONSTANTS ARE IN REALITY
C     BINARY MASKS WHICH ARE USED AS THE CRUCIAL KLUDGE WITHIN THIS
C     ROUTINE
C     THE VALUES ARE, IN ORDER, X'B0', X'B000', X'B00000', AND
C     X'B0000000'.
      DATA IFAKE/ 176, 45056, 11534336, -1342177280/
      EQUIVALENCE (ALPHA,IALPH)
      ALPHA=GGGG
      I=II
      N=0
   10 N=N+1
      K=MOD(I,10)
C     SHIFTS 'N-1' BYTES LEFT.
      K=K*2**(8*(N-1))
      IALPH=IALPH+K+IFAKE(N)
      I=I/10
      IF (I.NE.0) GO TO 10
      ANUMB=ALPHA
      RETURN
      END
```

```
      SUBROUTINE PRINT1(A,NV,LABELS)
C
C     SUBROUTINE PRINT1 FOR 1 DIMENSIONAL ARRAY
C     A IS ARRAY TO BE PRINTED
C     NV IS NUMBER OF VALUES TO BE PRINTED
C     LABELS IS ARRAY OF LABELS FOR ARRAY A
C     * NOTE * THIS ROUTINE MUST BE USED WITH SUBROUTINE RDLBL
C
      REAL*8 NN,LABELS
      DIMENSION A(1),LABELS(1),NN(8)
      COMMON /IO/LPD,LPU,LPR
      IT=1
      KK=0
      K1=1
      K2=MIN0(8,NV)
    1 KK=KK+8
      IF (NV-KK) 5,5,3
    3 IT=IT+1
      GO TO 1
    5 DO 10 JX=1,IT
      LLL=K2-K1+1
      LL=0
      DO 20 JJ=K1,K2
      LL=LL+1
   20 NN(LL)=LABELS(JJ)
      WRITE (LPR,100) (NN(II),II=1,LLL)
      WRITE (LPR,101) (A(I),I=K1,K2)
      K1=K2+1
      K2=K1+7
      K2=MIN0(K2,NV)
   10 CONTINUE
  100 FORMAT (///,13X,A6,7(9X,A6))
  101 FORMAT (1H0,7X,8E15.6)
      RETURN
      END
```

```
      SUBROUTINE PRINT2(A,N,LABELS)
C
C     SUBROUTINE PRINT2 FOR PRINTING OF LABELED SYMMETRIC MATRICES
C     A IS ARRAY TO BE PRINTED
C     N IS NUMBER OF VALUES TO BE PRINTED
C     LABELS IS ARRAY OF LABELS FOR TITLING OF ARRAY A
C     * NOTE * THIS ROUTINE MUST BE USED WITH SUBROUTINE RDLBL
C
      REAL*8 NN,LABELS
      DIMENSION A(100,100), LABELS(1), NN(8)
      COMMON /IO/LRD,LPU,LPR
      IT=1
      KK=0
      K1=IT
      K2=MIN0(8,N)
    1 KK=KK+8
      IF (N-KK) 5,5,3
    3 IT=IT+1
      GO TO 1
    5 DO 10 JX=1,IT
      LLL=K2-K1+1
      LL=0
      DO 20 JJ=K1,K2
      LL=LL+1
   20 NN(LL)=LABELS(JJ)
      WRITE (LPR,100) (NN(II),II=1,LLL)
      WRITE (LPR,102)
      DO 30 I=1,N
   30 WRITE (LPR,101) LABELS(I),(A(I,J),J=K1,K2)
      K1=K2+1
      K2=K1+7
      K2=MIN0(K2,N)
   10 CONTINUE
  100 FORMAT (///,13X,A6,7(9X,A6))
  101 FORMAT (1H ,A6,1X,8E15.5)
  102 FORMAT (1H )
      RETURN
      END
```

```
      SUBROUTINE PRINT3(A,NC,NR,LABELS,JK)
C
C     SUBROUTINE PRINT3 FOR PRINTING LABELED ARRAYS
C     A IS ARRAY TO BE PRINTED
C     NC IS NUMBER OF COLUMNS
C     NR IS NUMBER OF ROWS
C     LABELS IS ARRAY OF LABELS FOR TITLING OF ARRAY A
C     * NOTE * THIS ROUTINE MUST BE USED WITH SUBROUTINE RDLBL
C     JK IS INDICATOR
C        0 = PRINTS ENTIRE ARRAY
C        N = PRINTS THE N(TH) ROW OF ARRAY A
C
      REAL*8 NN,LABELS
      DIMENSION A(50,100), LABELS(1), NN(8)
      COMMON /IO/LRD,LPU,LPR
      IT=1
      KK=0
      K1=IT
      K2=MIN0(8,NC)
    1 KK=KK+8
      IF (NC-KK) 5,5,3
    3 IT=IT+1
      GO TO 1
    5 DO 10 JX=1,IT
      LLL=K2-K1+1
      LL=0
      DO 20 JJ=K1,K2
      LL=LL+1
   20 NN(LL)=LABELS(JJ)
      WRITE (LPR,100) (NN(II),II=1,LLL)
      WRITE (LPR,102)
      IF (JK .NE. 0) GO TO 35
      DO 30 I=1,NR
   30 WRITE (LPR,101) I,(A(I,J),J=K1,K2)
      GO TO 40
   35 WRITE (LPR,101)JK,(A(JK,J),J=K1,K2)
   40 K1=K2+1
      K2=K1+7
      K2=MIN0(K2,NC)
   10 CONTINUE
  100 FORMAT (///,13X,A6,7(9X,A6))
  101 FORMAT (1H ,I5,1X,8E15.5)
  102 FORMAT (1H )
      RETURN
      END
```

APPENDIX B

TABLE B-1

REFLECTANCES OF 12 GRAY OBJECT COLORS METAMERIC WITH RESPECT TO THE 1931 CIE STANDARD OBSERVER AND CIE SOURCE C (WYSZECKI 1962)

WAVELENGTH (NM)	1	2	3	4	5	6
380	.0575	.0437	.1613	.1148	.0243	.2550
390	.2017	.1081	.1770	.1170	.0414	.2919
400	.3510	.1766	.2173	.1243	.0608	.3150
410	.4419	.2279	.2579	.1257	.0819	.3268
420	.4762	.2590	.2895	.1301	.1014	.3314
430	.4617	.2834	.3064	.1330	.1298	.3327
440	.4197	.2964	.3126	.1342	.1565	.3297
450	.3529	.3004	.3089	.2992	.2858	.3230
460	.2744	.3073	.2961	.4887	.4389	.3121
470	.1617	.3023	.2595	.4770	.4637	.2877
480	.0851	.3073	.2350	.3742	.4341	.2478
490	.0634	.3285	.3426	.2607	.3983	.2012
500	.0637	.3535	.4321	.2874	.3796	.1634
510	.0653	.3708	.4478	.4496	.3649	.1210
520	.0664	.3769	.4269	.4852	.3555	.0654
530	.0677	.3664	.3798	.4136	.3397	.2039
540	.1404	.3354	.3233	.3123	.3098	.4011
550	.5696	.2851	.2707	.2222	.2643	.3582
560	.6792	.2147	.2275	.1676	.2002	.2929
570	.6046	.1465	.1985	.1558	.1362	.3819
580	.4811	.0890	.1812	.1893	.0818	.5962
590	.3513	.2425	.1702	.2394	.2734	.5061
600	.2360	.6854	.1647	.2979	.7078	.3429
610	.1433	.5856	.1705	.3520	.5950	.2065
620	.0809	.3676	.3035	.4021	.3832	.1179
630	.0422	.2145	.7796	.4401	.2282	.0626
640	.0188	.1177	.8000	.4737	.1259	.0291
650	.0094	.0589	.6492	.5029	.0629	.0128
660	.0031	.0294	.4984	.5249	.0315	.0074
670	.0016	.0126	.3924	.5424	.0157	.0083
680	.0000	.0042	.3117	.5578	.0079	.0113
690	.0000	.0000	.2694	.5724	.0039	.0181
700	.0059	.0019	.2530	.5812	.0000	.0223
710	.0188	.0061	.2497	.5885	.0000	.0231
720	.0282	.0092	.2438	.5958	.0000	.0223
730	.0493	.0161	.2394	.6024	.0000	.0210
740	.0634	.0206	.2329	.6089	.0000	.0202
750	.0728	.0237	.2297	.6149	.0000	.0189
760	.0821	.0268	.2305	.6199	.0000	.0181

TABLE B-1

REFLECTANCES OF 12 GRAY OBJECT COLORS METAMERIC WITH RESPECT TO THE 1931 CIE STANDARD OBSERVER AND CIE SOURCE C (WYSZECKI 1962)

WAVELENGTH (NM)	7	8	9	10	11	12
380	.6061	.2190	.2706	.2411	.3749	.1042
390	.6017	.2208	.2945	.3158	.3781	.1101
400	.5503	.2208	.3096	.3703	.3770	.1168
410	.4611	.2225	.3172	.4026	.3715	.1220
420	.3748	.2243	.3202	.4167	.3655	.1299
430	.3230	.2261	.3210	.4106	.3569	.1364
440	.2986	.2296	.3191	.3915	.3451	.1443
450	.2890	.2350	.3148	.3521	.3299	.1563
460	.2847	.2668	.3077	.2906	.3086	.2258
470	.2825	.3985	.2918	.1933	.2711	.5199
480	.2816	.5283	.2685	.0960	.2184	.8121
490	.2812	.4980	.2408	.0494	.1614	.7539
500	.2812	.4023	.2171	.0644	.1197	.5457
510	.2811	.3528	.1954	.2878	.0737	.3787
520	.2830	.2454	.1593	.5860	.0433	.2799
530	.3029	.3171	.1475	.5752	.0553	.2296
540	.3335	.2877	.2315	.4341	.1910	.2141
550	.3254	.2620	.4136	.2822	.4895	.2048
560	.3094	.2433	.5146	.1664	.6621	.2029
570	.2971	.2322	.4711	.0904	.6002	.2165
580	.2895	.2253	.3723	.0472	.4536	.2485
590	.2849	.2222	.2813	.2467	.3176	.2839
600	.2822	.2206	.2143	.6708	.2161	.3208
610	.2810	.2251	.1727	.5661	.1520	.3587
620	.2815	.3164	.1446	.3673	.1164	.3937
630	.2915	.6399	.1326	.2175	.0956	.4207
640	.3394	.6538	.1729	.1200	.0912	.4477
650	.3568	.5513	.4231	.0600	.0871	.4707
660	.2490	.4488	.6223	.0300	.0878	.4888
670	.3385	.3768	.6528	.0150	.0953	.5044
680	.3314	.3215	.6163	.0075	.1310	.5180
690	.3443	.2882	.5720	.0037	.2155	.5298
700	.3938	.2633	.5265	.0000	.3231	.5381
710	.4556	.2467	.4826	.0000	.3886	.5455
720	.5062	.2356	.4422	.0000	.4252	.5532
730	.5481	.2273	.4078	.0000	.4429	.5602
740	.5817	.2218	.3717	.0000	.4474	.5669
750	.5999	.2190	.3436	.0000	.4495	.5730
760	.5859	.2190	.3137	.0000	.4518	.5787

TABLE B-2

CIE/CRI STANDARD REFLECTANCE FUNCTIONS

WAVELENGTH NM.	1	2	3	4	5	6	7	8
360	.1160	.0530	.0580	.0570	.1430	.0790	.1500	.0750
365	.1360	.0550	.0590	.0590	.1870	.0810	.1770	.0780
370	.1590	.0590	.0610	.0620	.2330	.0890	.2180	.0840
375	.1900	.0640	.0630	.0670	.2690	.1130	.2930	.0900
380	.2190	.0700	.0650	.0740	.2950	.1510	.3790	.1040
385	.2390	.0790	.0680	.0830	.3060	.2030	.4590	.1290
390	.2520	.0890	.0700	.0930	.3100	.2650	.5240	.1700
395	.2560	.1010	.0720	.1050	.3120	.3390	.5460	.2400
400	.2560	.1110	.0730	.1160	.3130	.4100	.5510	.3190
405	.2540	.1160	.0730	.1210	.3150	.4640	.5550	.4160
410	.2520	.1180	.0740	.1240	.3190	.4920	.5590	.4620
415	.2480	.1200	.0740	.1260	.3220	.5090	.5600	.4820
420	.2440	.1210	.0740	.1280	.3260	.5170	.5610	.4900
425	.2400	.1220	.0730	.1310	.3300	.5240	.5580	.4880
430	.2370	.1220	.0730	.1350	.3340	.5310	.5560	.4820
435	.2320	.1220	.0730	.1390	.3390	.5380	.5510	.4730
440	.2300	.1230	.0730	.1460	.3460	.5440	.5440	.4620
445	.2260	.1240	.0730	.1510	.3520	.5510	.5370	.4500
450	.2250	.1270	.0740	.1610	.3600	.5560	.5220	.4390
455	.2220	.1280	.0750	.1720	.3690	.5560	.5060	.4260
460	.2200	.1310	.0770	.1860	.3910	.5540	.4880	.4130
465	.2180	.1340	.0800	.2050	.3940	.5490	.4690	.3970
470	.2160	.1380	.0850	.2290	.4030	.5410	.4480	.3820
475	.2140	.1430	.0940	.2540	.4100	.5310	.4290	.3660
480	.2140	.1500	.1090	.2810	.4150	.5190	.4080	.3520
485	.2140	.1590	.1260	.3080	.4180	.5040	.3850	.3370
490	.2160	.1740	.1480	.3320	.4190	.4880	.3630	.3250
495	.2180	.1900	.1720	.3520	.4170	.4690	.3410	.3100
500	.2230	.2070	.1980	.3700	.4130	.4500	.3240	.2990
505	.2250	.2250	.2210	.3830	.4090	.4310	.3110	.2890
510	.2260	.2420	.2410	.3900	.4030	.4140	.3010	.2830
515	.2260	.2530	.2600	.3940	.3960	.3950	.2910	.2760
520	.2250	.2600	.2780	.3950	.3890	.3770	.2930	.2700
525	.2250	.2640	.3020	.3920	.3810	.3580	.2730	.2620
530	.2270	.2670	.3390	.3850	.3720	.3410	.2650	.2560
535	.2300	.2690	.3700	.3770	.3630	.3250	.2600	.2510
540	.2360	.2720	.3920	.3670	.3530	.3090	.2570	.2500
[illegible]	.2450	.2760	.3990	.3540	.3420	.2930	.2570	.2510
[illegible]	[illegible]	.2820	.4000	.3410	.3310	.2770	.2590	.2510
[illegible]	.2620	.2890	.3930	.3270	.3200	.2650	.2600	.2580
[illegible]	.2720	.2970	.3800	.3120	[illegible]	.2500	[illegible]	.2640
[illegible]	.2830	[illegible]	.3650	.2960	[illegible]	[illegible]	[illegible]	.2690
[illegible]	.3030	.3020	.3490	.2930	.2840	.2340	.2560	.2720
[illegible]	[illegible]	[illegible]	[illegible]	.2820	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2790
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2840
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2760	[illegible]
[illegible]	[illegible]	[illegible]	.2720	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-3

RECONSTRUCTED CIE/CRI REFLECTANCE FUNCTIONS: 3 COMPONENT

WAVELENGTH (NM)	1	2	3	4	5	6	7	8
360	.0950	.0805	.0317	.0796	.1160	.1073	.1233	.0776
365	.1098	.0937	.0212	.0935	.1422	.1227	.1437	.1051
370	.1279	.1076	.0105	.1086	.1731	.1455	.1722	.1195
375	.1497	.1192	-.0007	.1203	.2097	.1835	.2171	.1425
380	.1712	.1265	-.0081	.1332	.2458	.2303	.2701	.1870
385	.1892	.1280	-.0070	.1402	.2761	.2827	.3252	.2317
390	.2050	.1259	.0002	.1436	.3004	.3366	.3804	.2809
395	.2185	.1220	.0201	.1445	.3152	.3878	.4302	.3327
400	.2305	.1172	.0421	.1425	.3239	.4337	.4753	.3837
405	.2473	.1128	.0615	.1317	.3237	.4720	.5211	.4440
410	.2547	.1102	.0712	.1277	.3253	.4932	.5345	.4725
415	.2565	.1080	.0757	.1273	.3282	.5050	.5553	.4939
420	.2562	.1064	.0779	.1304	.3312	.5125	.5607	.4875
425	.2535	.1052	.0783	.1318	.3355	.5170	.5612	.4835
430	.2499	.1039	.0791	.1364	.3405	.5217	.5608	.4777
435	.2446	.1020	.0799	.1418	.3459	.5260	.5583	.4695
440	.2403	.1024	.0803	.1471	.3526	.5288	.5547	.4577
445	.2348	.1021	.0818	.1571	.3594	.5319	.5497	.4451
450	.2315	.1054	.0846	.1674	.3671	.5328	.5437	.4317
455	.2269	.1086	.0869	.1770	.3737	.5309	.5342	.4149
460	.2239	.1147	.0902	.1908	.3821	.5279	.5237	.3968
465	.2209	.1226	.0944	.2065	.3918	.5237	.5109	.3752
470	.2187	.1325	.1021	.2230	.3992	.5169	.4954	.3525
475	.2170	.1433	.1130	.2403	.4037	.5096	.4790	.3322
480	.2170	.1568	.1301	.2588	.4116	.5006	.4607	.3126
485	.2176	.1716	.1491	.2776	.4146	.4886	.4398	.2922
490	.2214	.1894	.1725	.2963	.4153	.4749	.4190	.2761
495	.2249	.2070	.1966	.3136	.4131	.4583	.3959	.2596
500	.2311	.2254	.2221	.3292	.4096	.4422	.3756	.2487
505	.2369	.2418	.2439	.3421	.4055	.4268	.3574	.2396
510	.2427	.2552	.2626	.3505	.3993	.4125	.3424	.2348
515	.2459	.2652	.2793	.3560	.3907	.3977	.3267	.2294
520	.2474	.2722	.2944	.3599	.3812	.3848	.3128	.2254
525	.2472	.2777	.3137	.3610	.3684	.3712	.2965	.2212
530	.2468	.2830	.3434	.3634	.3521	.3605	.2807	.2219
535	.2471	.2873	.3677	.3639	.3371	.3505	.2676	.2239
540	.2504	.2919	.3841	.3611	.3226	.3398	.2580	.2292
545	.2564	.2962	.3877	.3569	.3079	.3246	.2524	.2364
550	.2634	.3002	.3856	.3456	.2923	.3112	.2494	.2447
555	.2715	.3043	.3775	.3253	.2870	.2959	.2475	.2528
[illegible]	.2847	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2620
[illegible]	.2910	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2702
[illegible]	.3020	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2786
[illegible]	.3110	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2841
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3085	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-3

RECONSTRUCTED CIE/CRI REFLECTANCE FUNCTIONS: 3 COMPONENT
(CONTINUED)

WAVELENGTH NM	1	2	3	4	5	6	7	8
[illegible]	.4084	.3412	.2471	.1911	.2042	.1986	.3438	.4626
[illegible]	.4240	.3421	.2450	.1763	.1965	.2032	.3657	.4789
[illegible]	.4372	.3419	.2435	.1629	.1902	.2105	.3846	.5319
[illegible]	.4474	.3403	.2417	.1510	.1852	.2175	.4051	.5599
[illegible]	.4566	.3390	.2402	.1409	.1815	.2247	.4222	.5849
[illegible]	.4624	.3361	.2384	.1300	.1791	.2313	.4355	.6033
[illegible]	.4675	.3352	.2365	.1229	.1786	.2374	.4458	.6177
645	.4706	.3333	.2351	.1234	.1785	.2442	.4570	.6299
650	.4729	.3310	.2353	.1202	.1790	.2518	.4664	.6405
655	.4747	.3296	.2360	.1183	.1797	.2574	.4731	.6482
660	.4765	.3298	.2391	.1173	.1800	.2637	.4793	.6559
[illegible]	.4772	.3271	.2432	.1170	.1808	[illegible]	.4842	.6618
[illegible]	.4789	.3283	.2493	.1192	.1817	[illegible]	.4827	.6669
[illegible]	.4799	.3297	.2564	.1224	.1843	.2809	.4918	.6707
680	.4807	.3319	.2679	.1267	.1849	.2856	.4925	.6736
685	.4812	.3341	.2835	.1322	.1847	.2916	.4931	.6774
[illegible]	.4807	.3357	.3004	.1376	.1848	.2991	.4940	.6811
[illegible]	.4805	.3371	.3210	[illegible]	.1842	.3075	.4947	.6860
700	.4804	.3392	.3446	.1513	.1838	.3175	.4961	.6920
705	.4779	.3391	.3672	.1572	.1834	.3294	.4978	.6968
710	.4751	.3369	.3881	.1617	.1824	.3412	.5012	.7025
715	.4731	.3342	.4077	.1661	.1809	.3534	.5044	.7106
720	.4714	.3323	.4242	.1684	.1797	.3653	.5092	.7188
725	.4694	.3292	.4393	.1712	.1795	.3766	.5129	.7254
730	.4680	.3279	.4513	.1742	.1807	.3873	.5196	.7310
735	.4660	.3256	.4624	.1778	.1831	.3989	.5237	.7356
740	.4645	.3241	.4711	.1817	.1866	.4094	.5286	.7390
745	.4629	.3227	.4779	.1861	.1916	.4205	.5340	.7414
750	.4615	.3213	.4833	.1905	.1972	.4313	.5396	.7433
755	.4598	.3193	.4877	.1937	.2018	.4410	.5446	.7451
760	.4581	.3174	.4911	.1971	.2069	.4509	.5499	.7466
765	.4568	.3160	.4936	.2009	.2127	.4603	.5551	.7476
770	.4557	.3148	.4971	.2044	.2177	.4693	.5600	.7491
775	.4543	.3136	.5020	.2071	.2205	.4762	.5630	.7504
780	.4533	.3131	.5081	.2107	.2235	.4831	.5657	.7516
785	.4519	.3122	.5123	.2140	.2270	.4899	.5686	.7520
[illegible]	.4515	.3121	.5164	.2178	.2310	.4964	.5715	.7527
[illegible]	.4505	.3126	.5199	[illegible]	.2360	.5029	.5740	.7517
[illegible]	.4503	.3142	.5245	[illegible]	.2419	[illegible]	.5761	.7503
[illegible]	.4498	.3152	.5287	.2327	.2471	.5153	.5780	.7489
[illegible]	.4500	.3177	.5330	.2395	.2514	[illegible]	.5795	.7473
[illegible]	.4502	.3201	[illegible]	[illegible]	.2572	[illegible]	.5807	.7448
820	.4502	.3221	[illegible]	[illegible]	.2642	.5323	.5820	.7417
[illegible]	.4503	.3250			.2728	.5379	.5833	.7391
[illegible]	.4500	[illegible]			[illegible]	[illegible]	.5848	.7367

TABLE B-4

RECONSTRUCTED REFLECTANCE FUNCTIONS (ERROR), 3 COMPONENT

WAVELENGTH (NM)	1	2	3	4	5	6	7	8
360	.0210	-.0275	.0263	-.0226	.0270	-.0225	.0267	-.0226
365	.0262	-.0387	.0378	-.0345	.0448	-.0417	.0333	-.0271
370	.0311	-.0496	.0505	-.0466	.0599	-.0565	.0458	-.0355
375	.0403	-.0552	.0637	-.0553	.0593	-.0703	.0759	-.0585
380	.0478	-.0565	.0731	-.0592	.0492	-.0793	.1079	-.0930
385	.0498	-.0490	.0752	-.0572	.0299	-.0797	.1337	-.1027
390	.0470	-.0369	.0698	-.0506	.0096	-.0716	.1436	-.1109
395	.0375	-.0210	.0519	-.0395	-.0032	-.0489	.1158	-.0927
400	.0255	-.0062	.0309	-.0265	-.0109	-.0237	.0757	-.0647
405	.0067	.0032	.0115	-.0107	-.0087	-.0080	.0339	-.0280
410	-.0027	.0070	.0000	-.0039	-.0068	-.0012	.0145	-.0105
415	-.0085	.0120	-.0010	.0011	-.0062	.0020	.0047	-.0019
420	-.0121	.0146	-.0039	-.0024	-.0053	.0045	.0003	[illegible]
425	-.0135	.0168	-.0053	-.0005	-.0055	.0070	-.0032	.0045
430	-.0129	.0181	-.0061	-.0014	-.0065	.0093	-.0048	.0043
435	-.0126	.0201	-.0069	-.0020	-.0049	.0120	-.0073	.0045
440	-.0103	.0206	-.0077	-.0051	-.0066	.0152	-.0107	.0047
445	-.0088	.0219	-.0090	-.0061	-.0074	.0191	-.0149	[illegible]
450	-.0065	.0216	-.0106	-.0064	-.0071	.0232	-.0217	.0073
455	-.0049	[illegible]	-.0119	-.0059	-.0047	[illegible]	-.0272	.0111
460	-.0039	.0162	-.0131	-.0048	[illegible]	.0261	-.0337	.0162
465	-.0029	.0114	-.014?	-.0015	[illegible]	.0253	-.0410	[illegible]
470	-.0027	.0055	-.0171	.0060	[illegible]	.0241	-.0474	[illegible]
475	-.0030	-.0003	-.0190	.0138	.0033	.0213	-.0500	.0138
480	-.0030	-.0060	-.0211	.0222	.0034	.0184	-.0527	.0394
485	-.0036	-.0126	-.0231	.0304	.0034	.0154	-.0543	.0448
490	-.0054	-.0154	-.0245	.0357	.0037	.0131	-.0560	.0489
495	-.0069	-.0170	-.0246	.0384	.0039	.0107	-.0549	.0504
500	-.0081	-.0184	-.0241	.0408	.0034	.0078	-.0516	.0503
505	-.0119	-.0168	-.0229	.0409	.0035	.0042	-.0464	.0494
510	-.0167	-.0132	-.0216	.0395	.0037	.0015	-.0414	.0482
515	-.0199	-.0122	-.0193	.0390	.0053	-.0027	-.0357	.0466
520	-.0224	-.0122	-.0164	.0361	.0078	-.0078	-.0298	.0445
525	-.0222	-.0137	-.0117	.0310	.0126	-.0132	-.0235	[illegible]
530	-.0198	-.0160	-.0041	.0216	.017?	-.0195	-.0157	.0341
535	-.0171	-.0182	.0023	.0130	.0259	-.0255	-.0076	.0271
[illegible]	-.0144	-.0179	.0079	.0059	.0304	-.0298	-.0010	.0203
[illegible]	-.0116	[illegible]	.0118	[illegible]	.0321	[illegible]	.0046	.0136
[illegible]	-.0104	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0072
[illegible]	-.009?	-.0153	.011?	[illegible]	.0?10	[illegible]	.01??	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0012
[illegible]	-.0040	[illegible]	[illegible]	-.0229	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-4

RECONSTRUCTED REFLECTANCE FUNCTIONS (ERROR): 3 COMPONENT
(CONTINUED)

WAVELENGTH (NM)	1	2	3	4	5	6	7	8
610	.0336	.0008	.0049	-.0221	-.0172	.0214	.0002	-.0286
615	.0240	-.0012	.0015	-.0127	-.0075	.0161	-.0037	-.0169
620	.0128	-.0009	-.0028	-.0027	-.0052	.0125	-.0096	-.0039
625	.0036	-.0013	-.0067	.0090	-.0022	.0095	-.0161	.0081
630	-.0056	.0000	-.0112	.0131	-.0015	.0083	-.0222	.0191
635	-.0114	.0011	-.0144	.0188	-.0021	.0077	-.0255	.0257
640	-.0165	.0023	-.0165	.0231	-.0026	.0066	-.0268	.0303
645	-.0196	.0037	-.0191	.0256	-.0035	.0068	-.0280	.0331
650	-.0229	.0050	-.0193	.0270	-.0040	.0062	-.0284	.0355
655	-.0247	.0054	-.0200	.0297	-.0047	.0056	-.0281	.0368
660	-.0255	.0052	-.0201	.0307	-.0050	.0048	-.0273	.0371
665	-.0262	.0042	-.0199	.0314	-.0046	.0034	-.0273	.0382
670	-.0259	.0032	-.0193	.0312	-.0029	.0024	-.0269	.0381
675	-.0259	.0013	-.0184	.0316	-.0013	.0001	-.0258	.0383
680	-.0257	-.0009	-.0169	.0311	.0011	-.0026	-.0245	.0384
685	-.0242	-.0041	-.0148	.0298	.0043	-.0056	-.0231	.0376
690	-.0227	-.0063	-.0124	.0274	.0072	-.0081	-.0210	.0359
695	-.0205	-.0091	-.0090	.0240	.0109	-.0115	-.0177	.0330
700	-.0184	-.0112	-.0046	.0197	.0152	-.0155	-.0131	.0290
705	-.0149	-.0114	.0012	.0132	.0166	-.0164	-.0088	.0232
710	-.0111	-.0101	.0019	.0073	.0166	-.0162	-.0042	.0166
715	-.0081	-.0092	.0043	.0019	.0171	-.0154	-.0014	.0114
720	-.0054	-.0083	.0062	-.0026	.0163	-.0143	.0019	.0062
725	-.0034	-.0058	.0077	-.0072	.0155	-.0126	.0042	.0016
730	-.0020	-.0039	.0087	-.0102	.0143	-.0113	.0064	-.0020
735	.0000	-.0026	.0096	-.0128	.0129	-.0099	.0083	-.0056
740	.0025	-.0021	.0099	-.0137	.0104	-.0084	.0104	-.0090
745	.0041	-.0017	.0101	-.0141	.0094	-.0075	.0120	-.0114
750	.0055	-.0013	.0097	-.0135	.0058	-.0063	.0134	-.0133
755	.0072	-.0013	.0093	-.0127	.0032	-.0050	.0144	-.0151
760	.0089	-.0014	.0089	-.0121	.0011	-.0039	.0151	-.0166
765	.0102	-.0010	.0084	-.0119	-.0007	-.0023	.0149	-.0176
770	.0113	.0002	.0079	-.0124	-.0027	-.0003	.0150	-.0191
775	.0127	.0004	.0080	-.0131	-.0035	.0008	.0150	-.0204
780	.0137	.0009	.0079	-.0137	-.0045	.0019	.0153	-.0216
785	.0151	.0008	.0077	-.0140	-.0050	.0031	.0144	-.0220
790	.0155	.0004	.0074	-.0138	-.0050	.0036	.0135	-.0217
795	.0155	-.0006	.0071	[illegible]	-.0050	.0031	.0130	-.0207
800	.0157	-.0022	.0065	-.0105	[illegible]	.0027	.0119	-.0193
805	.0162	-.0033	.0063	-.0089	-.0041	.0017	.0110	-.0179
[illegible]	.0160	-.0043	.0060	[illegible]	-.0071	[illegible]	.0115	-.0163
[illegible]	.0155	-.0091	.0057	[illegible]	[illegible]	-.0014	.0091	-.0138
[illegible]	.0148	-.0119	.0050	[illegible]	.0112	-.0053	.0080	-.0107
[illegible]	.0147	[illegible]	.0040	[illegible]	[illegible]	[illegible]	.0072	-.0081
[illegible]	.0138	-.0178	[illegible]	[illegible]	.0160	[illegible]	.0071	-.0057

TABLE B-5

RECONSTRUCTED CIE/CPT REFLECTANCE FUNCTIONS: 7 COMPONENT

WAVELENGTH	1	2	3	4	5	6	7	8
360	.1160	.0530	.0580	.0570	.1420	.0790	.1500	.0750
365	.1360	.0550	.0590	.0590	.1370	.0810	.1770	.0790
370	.1590	.0570	.0610	.0620	.2230	.0890	.2180	.0840
375	.1900	.0640	.0630	.0670	.2590	.1130	.2930	.0900
[illegible]	.2190	.0700	.0650	.0740	.2950	.1510	.3780	.1040
385	.2390	.0790	.0680	.0820	.3040	.2031	.4590	.1290
390	.2520	.0887	.0699	.0930	.3100	.2651	.5241	.1700
395	.2560	.1010	.0720	.1050	.3120	.3390	.5460	.2400
400	.2560	.1110	.0730	.1160	.3130	.4100	.5510	.3190
405	.2540	.1160	.0730	.1210	.3150	.4640	.5550	.4160
[illegible]	.2520	.1190	.0740	.1240	.3190	.4920	.5590	.4620
[illegible]	.2490	.1200	.0740	.1260	.3210	.5080	.5600	[illegible]
[illegible]	.2440	.1210	.0740	.1300	.3260	.5170	.5610	[illegible]
[illegible]	.2400	.1220	.0770	[illegible]	.3300	.5240	.5579	[illegible]
[illegible]	.2370	.1220	.0730	.1351	.3340	.5310	.5559	.4219
435	.2320	.1220	.0730	.1391	.3390	.5380	.5509	.4729
[illegible]	.2300	.1230	.0730	.1431	.3480	.5439	.5439	[illegible]
[illegible]	.2260	.1240	.0730	[illegible]	.3520	.5509	.5349	.4479
[illegible]	.2230	.1270	.0740	[illegible]	.3600	.5560	.5220	.4390
455	.2220	.1290	.0750	[illegible]	.3690	.5560	.5060	.4260
[illegible]	.2200	.1310	.0770	[illegible]	.3800	[illegible]	.4880	.4100
[illegible]	.2170	[illegible]	.0800	[illegible]	.3940	[illegible]	.4690	[illegible]
[illegible]	.2160	.1390	.0850	[illegible]	.4050	.5410	.4490	[illegible]
[illegible]	.2140	.1430	.0940	.2540	.4100	.5310	.4290	[illegible]
[illegible]	.2140	.1500	.1090	.2910	.4150	.5190	.4081	.3521
[illegible]	.2140	.1589	.1259	.3079	.4180	.5041	.3851	.3371
490	.2160	.1709	.1479	.3219	.4190	.4881	.3631	.3231
495	.2180	.1899	.1719	.3519	.4170	.4691	.3411	.3101
500	.2230	.2069	.1979	.3699	.4130	.4501	.3241	.2991
505	.2250	.2249	.2209	.3829	.4090	.4311	.3111	.2991
510	.2260	.2419	.2409	.3900	.4030	.4141	.3011	[illegible]
515	.2260	.2530	.2600	.3940	.3960	.3950	.2910	.2760
520	.2250	.2600	.2780	[illegible]	[illegible]	.3770	.2830	.2700
525	.2250	.2640	.3000	.3920	.3810	.3580	.2730	.2620
530	.2270	.2670	.3390	.3950	.3720	.3410	.2650	[illegible]
[illegible]	.2300	[illegible]	.3700	[illegible]	.3630	.3250	.2600	[illegible]
[illegible]	.2360	.2720	.3920	[illegible]	[illegible]	.3090	.2570	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2570	.2510
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.2540
[illegible]	.2610	[illegible]	[illegible]	[illegible]	[illegible]	.2650	[illegible]	.2530
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
							[illegible]	[illegible]
	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]				[illegible]	[illegible]	[illegible]	.2931
	[illegible]				[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]				[illegible]	[illegible]	[illegible]	[illegible]
					[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-5

RECONSTRUCTED DIFFUSE REFLECTANCE FUNCTIONS: 7 COMPONENT
(CONTINUED)

WAVELENGTH (NM)	1	2	3	4	5	6	7	8
[illegible]	.4420	.3420	.2520	[illegible]	.1940	.2200	.3341	.4341
[illegible]	.4490	[illegible]	.2470	.1630	.1890	.2200	.3620	.4921
[illegible]	.4500	.3410	.2410	.1600	.1850	.2230	.3770	.5280
[illegible]	.4510	.3390	.2350	.1560	.1830	.2270	[illegible]	.5690
[illegible]	.4510	.3370	.2290	.1540	.1800	.2330	.4000	.6040
[illegible]	.4510	.3350	.2240	.1520	.1770	.2390	.4100	.6290
[illegible]	.4510	.3380	.2200	.1510	.1760	.2440	.4200	.6480
[illegible]	.4510	.3370	.2170	.1490	.1750	.2510	.4290	.6630
650	.4500	.3360	.2160	.1480	.1750	.2580	.4380	.6760
655	.4500	.3350	.2160	.1480	.1750	.2630	.4450	.6850
[illegible]	.4510	.3340	.2190	[illegible]	.1750	.2680	.4520	.6920
[illegible]	.4510	[illegible]	.2240	[illegible]	.1770	.2730	.4570	.7000
[illegible]	.4530	[illegible]	.2300	[illegible]	.1800	.2780	.4620	.7050
[illegible]	.4540	.3310	.2380	[illegible]	.1830	.2810	.4650	.7090
[illegible]	.4550	.3310	.2510	.1520	.1860	.2830	.4680	.7120
[illegible]	.4570	[illegible]	.2690	.1610	.1890	.2860	.4700	.7150
[illegible]	.4580	.3290	.2890	.1600	.1920	.2910	.4730	.7170
[illegible]	.4600	.3270	.3120	.1680	.1950	.2950	.4770	.7190
700	.4620	.3290	.3400	.1700	.1990	.3020	.4830	.7210
[illegible]	.4630	.3270	.3660	.1710	.2000	.3130	.4890	.7220
[illegible]	.4640	.3260	.3900	[illegible]	.1990	.3250	.4960	[illegible]
[illegible]	.4650	[illegible]	.4120	.1680	.1980	.3380	.5030	[illegible]
[illegible]	.4660	.3250	.4310	.1660	.1960	.3510	.5110	.7250
[illegible]	.4660	.3240	.4470	.1640	.1950	.3640	.5180	.7270
[illegible]	.4660	.3240	.4600	.1640	.1950	.3760	.5250	.7290
735	.4660	.3230	.4720	.1650	.1960	.3890	.5320	.7300
740	.4670	.3220	.4810	.1680	.1970	.4010	.5390	.7300
745	.4670	.3210	.4890	.1720	.2000	.4130	.5460	.7300
750	.4670	.3200	.4930	.1770	.2030	.4250	.5530	.7300
755	.4670	.3180	.4970	.1810	.2050	.4360	.5590	.7300
760	.4670	.3160	.5000	.1850	.2080	.4470	.5650	.7300
765	.4670	.3150	.5020	.1890	.2120	.4580	.5700	.7300
770	.4670	.3150	.5050	.1920	.2150	.4690	.5750	.7300
775	.4670	.3140	.5100	.1940	.2170	.4770	.5780	.7300
780	.4670	.3140	.5160	.1970	.2190	.4850	.5810	.7300
[illegible]	.4670	.3130	.5200	.2000	.2220	.4930	.5830	.7300
790	.4670	.3130	.5240	.2040	.2230	.5000	.5850	.7310
795	.4660	[illegible]	.5270	[illegible]	.2310	.5060	.5870	.7310
[illegible]	.4660	[illegible]	[illegible]	[illegible]	.2370	.5120	[illegible]	.7310
[illegible]	.4660	[illegible]	.5350	.2250	.2430	.5170	.5890	.7310
[illegible]	.4660	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.7310
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.7310
[illegible]	.4650	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.5901	.7311
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.7311
[illegible]	.4640	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.7311

TABLE B-6

RECONSTRUCTED REFLECTANCE FUNCTIONS (ERRORS) Z COMPONENT

EL	TH 1	2	3	4	5	6	7	8
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	-.0001	.0000	.0000
[illegible]	.0000	.0001	.0001	.0000	.0000	-.0001	-.0001	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
405	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
420	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
[illegible]	.0000	.0000	.0000	-.0001	.0000	.0000	.0001	.0001
[illegible]	.0000	.0000	.0000	-.0001	.0000	.0000	.0001	.0001
[illegible]	.0000	.0000	.0000	-.0001	.0000	.0000	.0001	.0001
[illegible]	.0000	.0000	.0000	-.0001	.0000	.0001	.0001	.0001
[illegible]	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
[illegible]	.0000	.0000	.000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	[illegible]	[illegible]	.0000	[illegible]	[illegible]	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0001	.0000	-.0001	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0001	.0000	-.0001	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0001	.0000	-.0001	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0001	.0000	-.0001	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0001	.0000	-.0001	-.0001	-.0001
[illegible]	.0000	.0001	.0001	.0000	.0000	-.0001	-.0001	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	[illegible]	-.0001	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0001
[illegible]	.0000	[illegible]	[illegible]	[illegible]	.0000	.0000	[illegible]	.0001
[illegible]	.0000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-6

RECONSTRUCTED REFLECTANCE FUNCTIONS (ERROR): 7 COMPONENT
(CONTINUED)

WAVELENGTH	1	2	3	4	5	6	7	8
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	-.0001	-.0001
[illegible]	.0000	.0000	.0000	.0001	.0000	.0000	.0000	-.0001
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
650	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
735	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
740	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
745	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
750	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
760	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
765	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
770	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
775	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	[illegible]	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	[illegible]	.0000	.0000
[illegible]	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0000	.0000
[illegible]	.0000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0000	.0000
[illegible]	.0000	[illegible]	[illegible]	[illegible]	.0000	[illegible]	-.0001	-.0001
[illegible]	.0000	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0001
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-7

DATA MEAN AND COMPONENTS 1 - 7

WL STP (NM)	INPUT MEAN	COMPONENT 1	2	3	4	5	6	7
[illegible]	.0914	.0236	.0592	.0111	.0632	.0272	.0130	.0075
[illegible]	.1040	.0208	.0799	[illegible]	.0950	.0144	.0322	[illegible]
[illegible]	.1206	.0238	.1068	.0207	.1139	.0598	.0407	.0026
[illegible]	.1436	.0428	.1469	.0293	.1572	.0613	.0302	.0057
[illegible]	.1695	.0735	.1925	[illegible]	.1941	.0540	.0104	.0099
[illegible]	.1957	.1134	.2370	.1075	.2195	.0367	-.0157	.0118
[illegible]	.2216	.1593	.2779	.0993	.2175	.0165	-.0271	.0120
[illegible]	.2464	.2091	.3065	.0787	.1705	-.0020	-.0329	.0087
[illegible]	.2686	.2591	.3231	.0554	.1090	-.0156	-.0283	.0046
405	.2892	.3232	.3405	.0375	.0439	-.0113	-.0176	-.0025
410	.3000	.3530	.3496	.0277	.0139	-.0083	-.0123	-.0069
[illegible]	.3050	.3646	.3514	.0215	-.0017	-.0081	-.0100	-.0110
[illegible]	.3076	[illegible]	.3624	.0177	[illegible]	-.0076	-.0090	-.0144
[illegible]	.3082	[illegible]	.3682	.0140	-.0177	-.0096	-.0075	-.0162
[illegible]	.3087	.3516	.3747	.0113	-.0111	-.0124	-.0064	-.0163
[illegible]	.3084	.3431	.3817	.0068	-.0176	-.0155	-.0045	-.0174
[illegible]	.3092	.3205	.3874	.0045	-.0191	-.0190	-.0001	-.0172
[illegible]	.3077	.3116	.3933	.0006	-.0226	-.0230	.0032	-.0157
[illegible]	.3080	.2913	.3955	-.0112	-.0297	-.0256	.0083	-.0147
[illegible]	.3067	.2681	.3961	-.0027	-.0345	-.0242	.0146	-.0124
[illegible]	.3062	.2407	.3917	.0026	-.0411	-.0206	.0213	-.0101
[illegible]	.3057	[illegible]	.3920	-.0069	-.0479	-.0147	.0240	.0052
[illegible]	.3052	.1739	.3840	-.0102	-.0569	-.0078	.0265	.0011
[illegible]	.3051	.1357	.3727	.0044	-.0651	-.0006	.0250	.0079
[illegible]	.3060	.1029	.3545	-.0090	-.0697	.0073	.0231	.0153
[illegible]	.3064	.0655	.3317	-.0177	-.0759	.0150	.0206	.0221
490	.3081	.0309	.3024	-.0197	-.0806	.0205	.0183	.0242
495	.3086	-.0034	.2695	-.0242	-.0813	.0241	.0158	.0258
[illegible]	.3105	-.0317	.2347	-.0293	-.0794	.0273	.0120	.0268
505	.3117	-.0557	.2028	-.0328	-.0760	.0300	.0069	.0229
[illegible]	.3125	-.0717	.1736	-.0357	-.0726	.0311	.0021	.0168
515	.3114	-.0853	.1458	-.0399	-.0670	.0345	-.0014	.0126
[illegible]	.3096	-.0952	.1212	-.0455	-.0601	.0390	-.0041	.0087
[illegible]	.3071	-.1047	.0932	-.0559	-.0490	.0435	-.0030	.0054
[illegible]	.3065	-.1103	.0594	-.0753	-.0319	.0480	.0014	.0009
[illegible]	.3056	-.1134	.0304	-.0910	-.0152	.0515	.0049	-.0026
[illegible]	.3045	-.1106	.0029	[illegible]	-.0011	.0524	.0077	-.0052
[illegible]	.3021	[illegible]	-.0164	-.0941	.0101	.0520	.0092	-.0067
[illegible]	.2999	[illegible]	-.0301	[illegible]	[illegible]	.0592	.0066	-.0098
[illegible]	.2967	[illegible]	-.0465	[illegible]	.0221	[illegible]	.0098	-.0125
[illegible]	.2935	[illegible]	-.0520	-.0924	[illegible]	.0742	.0095	-.0162
[illegible]	.2876	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0192
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0212
[illegible]	.2835	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0169
[illegible]	.294	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.0133
[illegible]	.2807	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.2810	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.2825	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.2869	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	.2920	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE B-7

[illegible] MEAN AND COMPONENTS 1 - 7
(CONTINUED)

WAVELENGTH	INPUT MEAN	COMPONENT			4	5	6	7
[illegible]	.2996	[illegible]	-.1241	.1251	.0269	-.0409	.0205	.0143
[illegible]	.3066	.3052	-.1273	.1324	.0133	-.0275	.0154	.0122
[illegible]	.3131	.3253	-.1279	.1377	-.0035	-.0177	.0106	.0099
[illegible]	.3185	.3500	-.1262	.1414	-.0189	-.0048	.0083	.0074
[illegible]	.3237	.3604	-.1239	.1443	-.0347	.0022	.0014	.0047
[illegible]	.3275	.3802	-.1202	.1451	-.0448	.0067	-.0007	.0034
[illegible]	.3310	.4014	-.1140	.1476	-.0514	.0103	-.0032	.0017
[illegible]	.3340	.4160	-.1103	.1473	-.0567	.0112	-.0036	.0005
[illegible]	.3371	.429[illegible]	-.1060	.1457	-.0592	.0129	-.0082	-.0011
[illegible]	.3396	.4318	-.1013	.1442	-.0617	.0141	-.0104	-.0014
[illegible]	.3425	.4467	-.0990	.1409	-.0618	.0150	-.0119	-.0014
665	.3454	.4530	-.0962	.1359	-.0621	.0176	-.0119	-.0013
670	.3489	.45[illegible]	-.0956	.1316	-.0610	.0189	-.0115	-.0010
[illegible]	.3520	[illegible]	-.0980	.1240	-.0594	.0222	-.0114	-.0004
[illegible]	.3555	.4545	-.1000	.1176	-.0567	.0239	-.0106	-.0002
[illegible]	.3597	[illegible]	-.1052	.1071	-.0523	.0299	-.0084	.0008
[illegible]	.3641	.4507	-.1103	.0909	-.0469	.0326	-.0063	.0003
[illegible]	.3694	.4535	-.1172	.0727	-.0390	.0357	-.0037	.0008
[illegible]	.3756	.4471	-.1247	.0540	-.0226	.0393	-.0008	-.0009
[illegible]	.3811	.4450	-.1298	.0321	-.0121	.0366	.0015	-.0017
[illegible]	.3861	.4452	-.1318	.0114	-.0085	.0322	.0031	-.0028
[illegible]	.3914	.4502	-.1341	-.0077	-.0010	.0292	.0054	-.0043
[illegible]	.3962	[illegible]	[illegible]	-.0261	.0059	.0225	.0064	-.0054
[illegible]	.4006	[illegible]	[illegible]	[illegible]	.0115	.0182	.0074	-.0076
[illegible]	.4049	[illegible]	[illegible]	-.0590	.0157	.0141	.0074	-.0088
[illegible]	.4091	.4631	-.1284	-.0680	.0197	.0099	.0076	-.0091
[illegible]	.4131	.4698	-.1236	-.0752	.0230	.0058	.0065	-.0076
[illegible]	.4171	.4651	-.1168	-.0870	.0253	.0029	.0054	-.0066
750	.4210	.4647	-.1039	-.0944	.0265	-.0001	.0035	-.0049
755	.4241	.4648	-.1016	-.1013	.0275	-.0030	.0021	-.0027
760	.4272	.4647	-.0933	-.1074	.0282	-.0055	.0010	-.0007
765	.4304	.4638	-.0849	-.1122	.0280	-.0082	.0007	.0007
770	.4335	.4634	-.0777	-.1177	.0283	-.0118	.0003	.0011
775	.4359	.4629	-.0737	-.1240	.0292	-.0138	.0009	.0018
780	.4386	.4614	-.0706	-.1307	.0300	-.0159	.0009	.0022
785	.4410	.4600	-.0661	-.1362	.0299	-.0175	.0019	.0033
790	.4437	.4580	-.0622	-.1408	.0290	-.0178	.0026	.0038
795	.4462	.4541	-.0571	-.1443	.0279	-.0161	.0023	.0051
800	.4497	.4485	-.0531	-.1487	.0260	-.0147	.0023	.0071
[illegible]	.4521	.441[illegible]	-.0492	-.1534	.0249	-.0122	.0028	.0091
[illegible]	.4550	.4370	-.0462	-.1597	.0235	-.0096	.0025	.0107
[illegible]	.4584	.4291	-.0432	[illegible]	.0215	-.0071	.0028	.0131
[illegible]	.4616	[illegible]	-.0395	[illegible]	.0181	-.0105	.0022	.0153
[illegible]	.4647	[illegible]	[illegible]	[illegible]	[illegible]	-.0041	.0017	.0169
[illegible]	.4174	[illegible]	-.0331	[illegible]	.0147	.0092	.0006	.0201

a correlated color temperature of 6800 K and the CIE illuminant C as a reference was 80. Subsequent papers have shown an improvement of this figure to 94 when the three component distribution was combined with a continuous spectrum.[3]

Many questions have arisen concerning the adequateness of the original CIE set of eight color test samples for measurement of the general CRI. Nayatani et. al. showed that because of deficiencies in the number and choice of the test samples involved it is possible to derrive, for a given reference illuminant, an infinite number of test illuminants with a different SPD that all have perfect color rendering indices.[4] In agreeing with this view Dr. Wyszecki, of the National Research Council of Canada, has suggested the use of metameric gray object colors generated statistically so as to be non-linearly related in assessing the color matching properties of lamps.[5,6] Nayatani and Takahama have confirmed the effectiveness of using the twelve metameric grays derrived by Dr. Wyszecki in making an assessment of this type.[7]

Although the use of properly chosen line spectra can provide a wide gamut of colors these same spectra could be disastrous in reproducing certain colors. Since the CRI was shown to increase even more when combined SPDs were used it would be beneficial to know if the maximum of the three line spectra could be accounted for by an interrelationship in the test samples. If the samples are interrelated and can be represented by three functions it is highly probable that these three functions would be the cause of maximization of CRI at wavelengths of 450, 540, and 610 nm. Determination of the